Published by IOP Publishing for SISSA
Received: *October 6, 2008*
Accepted: *November 2, 2008*
Published: *November 21, 2008*

Two-loop QCD corrections to the heavy-to-light quark decay

R. Bonciani and A. Ferroglia

Institut für Theoretische Physik, Universität Zürich,
CH-8057 Zurich, Switzerland
E-mail: Roberto.Bonciani@physik.uzh.ch, Andrea.Ferroglia@physik.uzh.ch

Abstract: We present an analytic expression for the two-loop QCD corrections to the decay process $b \to u\,W^*$, where b and u are a massive and massless quark, respectively, while W^* is an off-shell charged weak boson. Since the W-boson can subsequently decay in a lepton anti-neutrino pair, the results of this paper are a first step towards a fully analytic computation of differential distributions for the semileptonic decay of a b-quark. The latter partonic process plays a crucial role in the study of inclusive semileptonic charmless decays of B-mesons. The three independent form factors characterizing the bWu vertex are provided in form of a Laurent series in $(d-4)$, where d is the space-time dimension. The coefficients in the series are expressed in terms of Harmonic Polylogarithms of maximal weight 4, and are functions of the invariant mass of the leptonic decay products of the W-boson.

Keywords: B-Physics, NLO Computations, Heavy Quark Physics, QCD.

JHEP11(2008)065

Contents

1. Introduction

The measurements of inclusive semileptonic B meson decays, such as $\bar{B} \to X_u\, l\, \overline{\nu}$ and $\bar{B} \to X_c\, l\, \overline{\nu}$, allow a precise determination of the CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$. The latter are relevant for the study of flavor and CP violation in the quark sector (for a recent review see [1]).

Total decay rates of the B meson are described by a local Operator Product Expansion (OPE) in inverse powers of the b-quark mass m_b. To leading order in $1/m_b$, the total B meson decay rate is equivalent to the decay rate of an on-shell b quark, which can be calculated in perturbation theory [2]. Many authors contributed to the calculation of the radiative corrections to the total decay rate of $b \to u\, l\, \overline{\nu}$ and $b \to c\, l\, \overline{\nu}$, at $\mathcal{O}(\alpha_S)$ [3, 4] and $\mathcal{O}(\alpha_S^2)$ [5 − 12]. However, experimental collaborations need to impose cuts (also severe) on the kinematic variables. For instance, in charmless semileptonic decays, the need to suppress the charm background (which is ~ 50 times larger than the signal) forces one to restrict the measurements to the "shape-function region", in which the hadronic final state has large energy ($E_X \sim m_b$), but only moderate invariant mass ($\sim m_b \Lambda_{\mathrm{QCD}}$). It is therefore of great interest to consider differential decay distributions, from which it is possible to derive predictions for partial decay rates with arbitrary cuts. In this context, a first important set of results was obtained in [13], where it is possible to find analytic expressions for the NLO triple-differential distribution of the semileptonic $\bar{B} \to X_u\, l\, \overline{\nu}$ decay together with several double and single differential distributions for the same process. The resummation of threshold logarithms to next-to-leading approximation in the $b \to u$

JHEP11(2008)065

transition was considered in [14]. Higher order contributions to $\bar{B} \to X_u \, l \, \overline{\nu}$ decays were considered in [15 − 18] and, very recently, the full NNLO QCD corrections to the partonic process $b \to c \, l \, \overline{\nu}$ were obtained in [19]. Since the OPE applies only to sufficiently inclusive quantities, different frameworks were developed in order to account for effects due to cuts on the kinematic space [20 − 26]. In particular, in the shape-function region, Soft Collinear Effective Theory (SCET) provides an appropriate framework for the evaluation of the triple-differential distribution of the inclusive semileptonic decay $\bar{B} \to X_u \, l \, \overline{\nu}$. The NLO analysis of the latter process within the SCET approach is presented in [22, 23]. At NNLO, the situation is more complicated, but the jet and soft functions are known to $\mathcal{O}(\alpha_S^2)$ in perturbation theory [27, 28]. The only missing piece is the hard function, which can be obtained from the two-loop QCD corrections to the decay of a b-quark into a u-quark and an off-shell W-boson [29]. On the other hand, these virtual corrections can be considered as a first step towards an exact evaluation of the NNLO QCD corrections to the heavy-to-light quark transition. To complete the latter calculation, it is also necessary to take into account the real emission.

In this work we focus on the calculation of the two-loop QCD corrections to the decay process $b \to u \, W^*$. We provide an analytic expression for the three independent vertex form factors characterizing the coupling of the quark current with the charged weak boson. These form factors are evaluated by employing a set of techniques which are by now standard in multiloop calculations (see for instance [30]). We generate the relevant Feynman diagrams with QGRAF [31]. The form factors are extracted directly from the Feynman diagrams by means of projector operators. The whole calculation is carried out in Dimensional Regularization (DR); UV and IR (soft and collinear) divergencies appear as poles in $(d-4)$, where d is the space-time dimension. Since we work in DR, a prescription for handling the matrix γ_5 in d-dimensions must be chosen. We employed a γ_5 which anticommutes with γ_μ in d-dimensions. This prescription is appropriate for the case under study, since it is known that the diagrams that we consider fulfill a canonical (non-anomalous) Ward identity. After applying the projectors, the contribution of individual Feynman diagrams to the form factors is given by a combination of dimensionally regularized scalar integrals. These integrals are related to a small set of master integrals (MIs) by means of the Laporta algorithm [32]. The MIs are evaluated by employing the Differential Equations method [33] and they are expressed as Laurent series in $(d - 4)$. The coefficients of the series are given in terms of Harmonic Polylogarithms (HPLs) [34] of a single dimensionless variable $y = q^2/m_b^2 = -M_l^2/m_b^2$, where q^2 is the squared momentum carried by the W-boson and M_l is the lepton pair invariant mass. Since y is negative in the physical region $(-1 \leq y \leq 0)$ we perform an analytic continuation $y \to -x - i0^+$, where now $x = M_l^2/m_b^2$, $0 \leq x \leq 1$. The analytic continuation is indeed completely trivial, since all the HPLs appearing in the result are real for $-1 \leq y \leq 0$. The form factors found with the above procedure still contain UV and IR divergencies. It is possible to get rid of the UV divergencies by means of the renormalization procedure. We renormalize the form factors in a mixed scheme: the heavy- and light-quark wave functions and the heavy-quark mass are renormalized in the *on-shell* (OS) scheme, while the strong coupling constant is renormalized in $\overline{\text{MS}}$ scheme. The results shown in this paper contain IR divergencies. In order to cancel them, it is necessary to combine these results with the appropriate jet and soft functions [22]. Analogously, one can add the exact real emission and consider physical observables which are sufficiently inclusive with respect to the hard and soft radiation.

The paper is structured as follows. In section 2, we introduce the Feynman diagrams involved in the calculation and we discuss their structure in terms of form factors. In section 3, it is possible to find the details of the UV renormalization procedure. In sections 4 and 5, we collect the analytic expressions of the UV renormalized one- and two-loop QCD corrections to the form factors, respectively. The expressions of the bare form factors as well as the contributions of the individual diagrams to the form factors can be found in [35]. In section 6, we discuss the Ward identity relevant for the $b \to u\, W^*$ decay and we prove that our form factors satisfy it. We also provide the analytic expression of the one- and two-loop QCD corrections to the scalar vertex in which the pseudo-Goldstone boson couples to the quarks, since it enters in the Ward identity fulfilled by the bWu vertex. Our conclusions can be found in section 7. Finally, in appendix A we collect the set of MIs employed in the calculation.

2. Feynman diagrams and form factors

We consider the decay process $b \to u\, W^* \to u\, l\, \bar{\nu}_l$. The bottom quark of mass m_b carries a momentum P and decays in an up-quark (considered as massless) which carries momentum p and a W-boson of momentum $q = P - p$. Subsequently, the W-boson decays in the pair $l\bar{\nu}_l$ of squared invariant mass $M_l^2 = -q^2$. The mass-shell conditions are such that $P^2 = -m_b^2$ and $p^2 = 0$. The Feynman diagrams contributing to the two-loop QCD corrections to the decay process $b \to u\, W^*$ are shown in figure 1. The most general vertex correction in the Standard Model can be described in terms of six form factors F_i and G_i $(i = 1, 2, 3)$:

$$V^\mu(P,p) = F_1(q^2)\gamma^\mu + \frac{1}{2m_b}F_2(q^2)\sigma^{\mu\nu}q_\nu + \frac{i}{2m_b}F_3(q^2)q^\mu + G_1(q^2)\gamma^\mu\gamma_5 + \frac{i}{2m_b}G_2(q^2)\gamma_5 q^\mu + \frac{i}{2m_b}G_3(q^2)\gamma_5\tilde{q}^\mu\,, \tag{2.1}$$

where $\tilde{q}_\mu = P_\mu + p_\mu$. The spinors $\overline{u}(p)$ and $u(P)$, multiplying eq. (2.1) from left and right, respectively, are not written down explicitly. We define $\sigma^{\mu\nu} = -i/2[\gamma^\mu, \gamma^\nu]$. Since the u-quark is taken as massless, only three of the above form factors are independent. By replacing

$$\overline{u}(p)\,[\gamma^\mu, \gamma^\nu]\,u(P)q_\nu = 2im_b\overline{u}(p)\gamma^\mu u(P) - 2\tilde{q}^\mu\overline{u}(p)u(P)\,, \tag{2.2}$$

in eq. (2.1), we find the following relations among the form factors in eq. (2.1):

$$F_2 = -G_3\,, \qquad F_3 = -G_2\,, \qquad F_1 + \frac{1}{2}F_2 = G_1\,. \tag{2.3}$$

Consequently, using the definitions[1] $P_L = (1+\gamma_5)/2$ and $P_R = (1-\gamma_5)/2$, we can rewrite the vertex structure as follows:

$$V^\mu(P,p) = 2G_1(q^2)\gamma^\mu P_L - \frac{i}{m_b}G_2(q^2)P_R q^\mu - \frac{i}{m_b}G_3(q^2)P_R\tilde{q}^\mu\,. \tag{2.4}$$

[1] We employ the notation and conventions of [44]. In particular, in our notation $\gamma_5 = -\gamma_5^{\rm bd}$, where $\gamma_5^{\rm bd}$ is the matrix commonly employed in the Bjorken-Drell notation.

JHEP11(2008)065

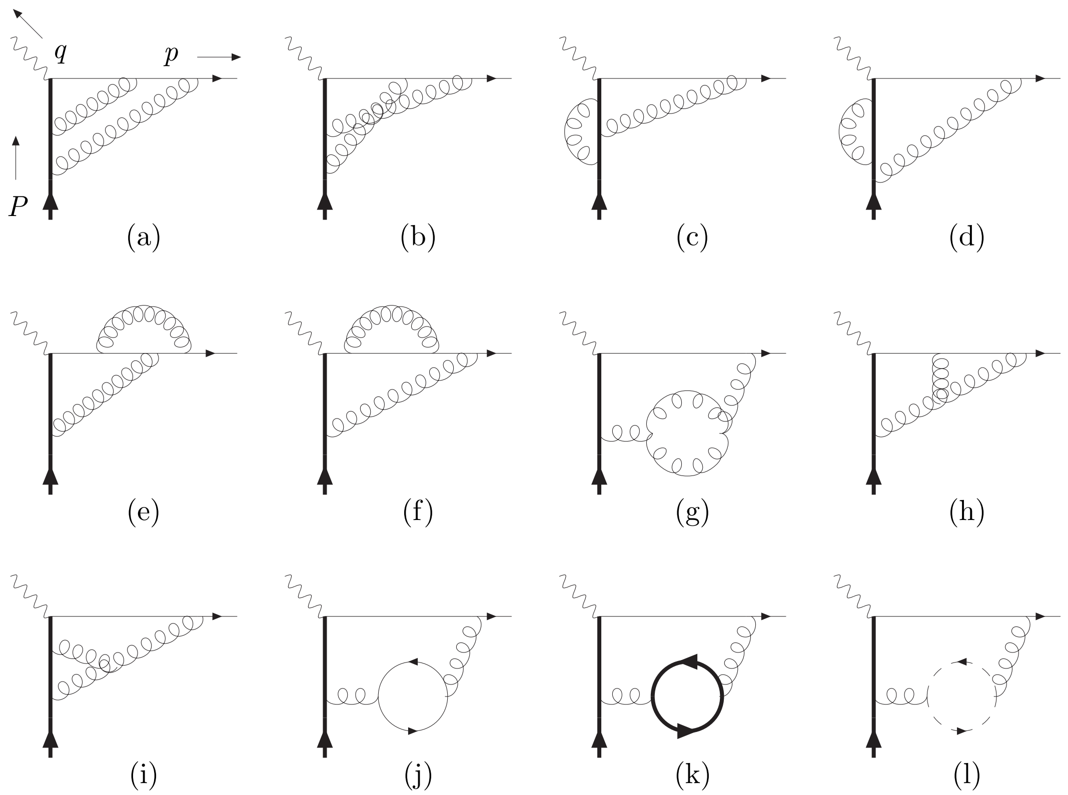


Figure 1: Feynman Diagrams for the Two-loop QCD corrections to the $b \to u\, W^*$ decay process.

The form factors are expanded in powers of α_s:

$$G_i = \frac{i g_w}{2\sqrt{2}} V_{ub} \left[G_i^{(0l)} + \left(\frac{\alpha_s}{\pi} \right) G_i^{(1l)} + \left(\frac{\alpha_s}{\pi} \right)^2 G_i^{(2l)} + \mathcal{O}\left(\frac{\alpha_s^3}{\pi^3} \right) \right] . \tag{2.5}$$

The purpose of the present work is to evaluate $G_i^{(2l)}$ where $i = 1, 2, 3$. V_{ub} represents the CKM matrix element and g_w the weak interaction coupling constant. With the normalization chosen in eq. (2.5), one finds

$$G_1^{(0l)} = 1\,, \qquad \text{and} \qquad G_2^{(0l)} = G_3^{(0l)} = 0\,. \tag{2.6}$$

The contribution of the virtual two-loop corrections to the hadronic tensor of [13] can be obtained by means of the following translation rules:

$$W_1^{(2l,\text{vir.})} = \frac{2}{m_b^2} \left[2 G_1^{(2l)} + \left(G_1^{(1l)} \right)^2 \right], \tag{2.7}$$

$$W_3^{(2l,\text{vir.})} = \frac{1-x}{4 m_b} \left(G_2^{(1l)} + G_3^{(1l)} \right)^2, \tag{2.8}$$

$$W_4^{(2l,\text{vir.})} = \frac{1}{m_b^2}\left\{G_3^{(2l)}+G_2^{(2l)}+G_1^{(1l)}\left(G_3^{(1l)}+G_2^{(1l)}\right)+\frac{1-x}{4}\left[\left(G_3^{(1l)}\right)^2-\left(G_2^{(1l)}\right)^2\right]\right\}, \quad (2.9)$$

$$W_5^{(2l,\text{vir.})} = \frac{2}{m_b^3}\left[G_3^{(2l)}-G_2^{(2l)}+G_1^{(1l)}\left(G_3^{(1l)}-G_2^{(1l)}\right)+\frac{1-x}{8}\left(G_3^{(1l)}-G_2^{(1l)}\right)^2\right]. \quad (2.10)$$

Consequently, the *hard functions* H_{ij}, as defined in [22], can be extracted using the relation between eqs. (16) and (17) of the same article. Note that the hadronic form factor W_2 does not receive contributions from two-loop virtual corrections.

We write our analytic results in terms of the dimensionless variable

$$x = -\frac{q^2}{m_b^2} = \frac{M_l^2}{m_b^2}, \quad 0 \le x \le 1 . \quad (2.11)$$

In writing or results, we employ the Harmonic Polylogarithms as defined in [34]; on top of the canonical weights, we introduce the weights -2 and 2, arising from the integrating factors $1/(x+2)$ and $1/(2-x)$, respectively. Actually, only two HPLs containing the weight 2 appear in the final result; they are

$$\begin{aligned} H(2;x) &= -\ln\left(1-\frac{x}{2}\right) = -H(-1,1-x)+\ln(2), \\ H(2,1,1;x) &= \int_0^x dt\frac{1}{2(2-t)}\ln^2(1-t) = \\ &= -\frac{1}{2}\ln(1-x)^2\ln(2-x)-\ln(1-x)\text{Li}_2(-1+x)+\text{Li}_3(-1+x)+\frac{3}{4}\zeta(3), \\ &= -H(-1,0,0,1-x)+\frac{3}{4}\zeta(3). \end{aligned} \quad (2.12)$$

For convenience, all the results of the paper, including the renormalized and bare form factors, as well as the contributions of individuals diagrams, are collected in the file `SemilepFF.txt` [35] included in the arXiv submission of the present work.

3. UV renormalization

The UV renormalization is performed by subtracting the one-loop sub-divergencies and the two-loop over-all divergencies. We renormalize the heavy- and light-quark wave functions and heavy-quark mass in the *on-shell* (OS) scheme, while the coupling constant α_S is renormalized in the $\overline{\text{MS}}$ scheme.

Neglecting for the time being mass renormalization, the bare and renormalized form factors satisfy the relation

$$G = Z_{2,u}^{\frac{1}{2}} Z_{2,b}^{\frac{1}{2}} G_{\text{bare}}(\alpha_s^{\text{bare}}), \quad (3.1)$$

where in the functions G we dropped the subscript $i = 1,2,3$.

The perturbative expansion of the various quantities in the equation above is

$$\begin{aligned} G &= \frac{ig_w}{2\sqrt{2}}V_{ub}\left(G^{(0l)}+aG^{(1l)}+a^2G^{(2l)}+\mathcal{O}\left(a_0^3\right)\right), \\ G_{\text{bare}} &= \frac{ig_w}{2\sqrt{2}}V_{ub}\left(G^{(0l)}+a_0G_{\text{bare}}^{(1l)}+a_0^2G_{\text{bare}}^{(2l)}+\mathcal{O}\left(a_0^3\right)\right), \end{aligned}$$

$$
\begin{aligned}
Z_{2,u} &= 1 + a_0 \delta Z_{2,u}^{(1l)} + a_0^2 \delta Z_{2,u}^{(2l)} + \mathcal{O}\left(a_0^3\right) , \\
Z_{2,b} &= 1 + a_0 \delta Z_{2,b}^{(1l)} + a_0^2 \delta Z_{2,b}^{(2l)} + \mathcal{O}\left(a_0^3\right) , \\
a_0 &= a \left(1 + a \delta Z_{\alpha}^{(1l)} + a^2 \delta Z_{\alpha}^{(2l)} + \mathcal{O}\left(a^3\right)\right) ,
\end{aligned} \tag{3.2}
$$

where we defined

$$
a_0 \equiv \frac{\alpha_s^{\rm bare}}{\pi} , \qquad a \equiv \frac{\alpha_s}{\pi} . \tag{3.3}
$$

Therefore, the one-loop renormalized amplitude is given by

$$
G^{(1l)} = G^{(1l)}_{\rm bare} + \frac{1}{2} \delta Z_{2,b}^{(1l)} G^{(0l)} , \tag{3.4}
$$

where we already took into account the fact that $\delta Z_{2,u}^{(1l)} = 0$ in the on-shell scheme. The two-loop renormalized amplitude reads instead

$$
\begin{aligned}
G^{(2l)} = & G^{(2l)}_{\rm bare} + \left[\frac{1}{2}\delta Z_{2,b}^{(2l)} + \frac{1}{2}\delta Z_{2,u}^{(2l)} + \frac{1}{2}\delta Z_{\alpha}^{(1l)}\delta Z_{2,b}^{(1l)} - \frac{1}{8}\left(\delta Z_{2,b}^{(1l)}\right)^2\right] G^{(0l)} \\
& + \left[\frac{1}{2}\delta Z_{2,b}^{(1l)} + \delta Z_{\alpha}^{(1l)}\right] G^{(1l)}_{\rm bare} .
\end{aligned} \tag{3.5}
$$

To account for mass renormalization, it is sufficient to add the contribution of the counter term diagram in figure 2 to the r. h. s. of the equation above.

The renormalization constants are the following:

$$
\delta Z^{(1l)}_{\alpha,\overline{\rm MS}}(d) = -C(d)\,\frac{1}{d-4}\left(-\frac{11}{6}C_A + \frac{1}{3}T_R(N_l + N_h)\right) , \tag{3.6}
$$

$$
\delta m^{(1l)}_{\rm OS}\left(d, m, \frac{\mu^2}{m^2}\right) = m\,C(d)\,\left(\frac{\mu^2}{m^2}\right)^{(4-d)/2}\,\frac{C_F}{2}\frac{(d-1)}{(d-4)\,(d-3)} , \tag{3.7}
$$

$$
\delta Z^{(1l)}_{2,b}\left(d, \frac{\mu^2}{m^2}\right) = C(d)\,\left(\frac{\mu^2}{m^2}\right)^{(4-d)/2}\,\frac{C_F}{2}\frac{(d-1)}{(d-4)\,(d-3)} , \tag{3.8}
$$

$$
\delta Z^{(2l)}_{2,u}\left(d, \frac{\mu^2}{m^2}\right) = C^2(d)\,\left(\frac{\mu^2}{m^2}\right)^{4-d}\frac{C_F}{8}N_h\left(-\frac{1}{2(d-4)} - \frac{5}{24}\right) , \tag{3.9}
$$

$$
\delta Z^{(2l)}_{2,b}\left(d, \frac{\mu^2}{m^2}\right) = C^2(d)\,\left(\frac{\mu^2}{m^2}\right)^{4-d}\,\frac{C_F}{2}\left[C_F f_1 + C_A f_2 + \frac{1}{2}N_l f_3 + \frac{1}{2}N_h f_4\right] , \tag{3.10}
$$

where μ is the renormalization scale and the constants $f_1, \cdots, f_4$ are [36]

$$
f_1 = \frac{9}{8(d-4)^2} - \frac{51}{32(d-4)} + \frac{433}{128} - \frac{3}{2}\zeta_3 - \pi^2\ln(2) - \frac{13}{16}\pi^2 + \mathcal{O}(d-4) , \tag{3.11}
$$

$$
f_2 = -\frac{11}{8(d-4)^2} + \frac{101}{32(d-4)} - \frac{803}{128} + \frac{3}{4}\zeta_3 - \frac{\pi^2}{2}\ln(2) + \frac{5}{16}\pi^2 + \mathcal{O}(d-4) , \tag{3.12}
$$

$$
f_3 = \frac{1}{2(d-4)^2} - \frac{9}{8(d-4)} + \frac{59}{32} + \frac{\pi^2}{12} + \mathcal{O}(d-4) , \tag{3.13}
$$

$$
f_4 = \frac{1}{(d-4)^2} - \frac{19}{24(d-4)} + \frac{1139}{288} - \frac{\pi^2}{3} + \mathcal{O}(d-4) . \tag{3.14}
$$

$$
\tag{3.15}
$$

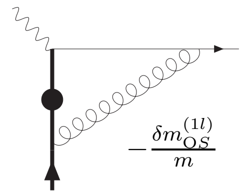


Figure 2: Mass-renormalization counter-term.

The factor $C(d)$ is

$$C(d) = (4\pi)^{(4-d)/2}\,\Gamma\left(3 - \frac{d}{2}\right) . \tag{3.16}$$

After UV renormalization, the vertex form factors still contain poles in $1/(d-4)$, which are associated to soft and collinear singularities.

4. One-loop form factors

In this section we collect the analytic expression of the one-loop renormalized form factors defined in eq. (2.5). In the formulas below, $C_F = (N_c^2 - 1)/2N_c$ is the Casimir operator of the fundamental representation of $\mathrm{SU}(N_c)$, where N_c is the number of colors (in the SM $N_c = 3$).

The form factor $G_1^{(1l)}$ is given by

$$G_1^{(1l)} = C(d)\left(\frac{\mu^2}{m^2}\right)^{\frac{4-d}{2}} C_F \sum_{i=-2}^{1} G_1^{(1l,i)}(d-4)^i + \mathcal{O}\left((d-4)^2\right) , \tag{4.1}$$

where the first four coefficients in the expansion in $(d-4)$ are

$$\begin{aligned}
G_1^{(1l,-2)} &= -1\,,\\
G_1^{(1l,-1)} &= \frac{5}{4} + H(1;x)\,,\\
G_1^{(1l,0)} &= -\frac{3}{2} + \frac{1-3x}{4x}H(1;x) - \frac{1}{2}H(0,1;x) - H(1,1;x)\,,\\
G_1^{(1l,1)} &= \frac{3}{2} - \frac{1-2x}{2x}H(1;x) - \frac{1-3x}{8x}H(0,1;x) - \frac{1-3x}{4x}H(1,1;x) + \frac{1}{4}H(0,0,1;x)\\
&\quad + \frac{1}{2}H(0,1,1;x) + \frac{1}{2}H(1,0,1;x) + H(1,1,1;x)\,.
\end{aligned} \tag{4.2}$$

The form factor $G_2^{(1l)}$ is

$$G_2^{(1l)} = C(d)\left(\frac{\mu^2}{m^2}\right)^{\frac{4-d}{2}} C_F \sum_{i=0}^{1} G_2^{(1l,i)}(d-4)^i + \mathcal{O}\left((d-4)^2\right) , \tag{4.3}$$

with

$$
\begin{aligned}
G_2^{(1l,0)} &= \frac{1}{x} - \frac{2-3x}{2x^2} H(1;x)\,, \\
G_2^{(1l,1)} &= -\frac{1}{x} + \frac{1-3x}{2x^2} H(1;x) + \frac{2-3x}{4x^2} H(0,1;x) + \frac{2-3x}{2x^2} H(1,1;x)\,.
\end{aligned} \tag{4.4}
$$

Finally, the form factor $G_3^{(1l)}$ is

$$
G_3^{(1l)} = C(d) \left(\frac{\mu^2}{m^2} \right)^{\frac{4-d}{2}} C_F \sum_{i=0}^{1} G_2^{(1l,i)} (d-4)^i + \mathcal{O}\left((d-4)^2 \right) \,, \tag{4.5}
$$

where

$$
\begin{aligned}
G_3^{(1l,0)} &= -\frac{1}{2x} H(1;x)\,, \\
G_3^{(1l,1)} &= \frac{1}{x} H(1;x) + \frac{1}{2x} H(1,1;x) + \frac{1}{4x} H(0,1;x)\,.
\end{aligned} \tag{4.6}
$$

Note that the IR poles of the form factor $G_1^{(1l)}$ exponentiate. This means that from the $\mathcal{O}(\alpha_S^2)$ expansion of the form factor

$$
\mathcal{F} = \exp\{G_1^{(1l)}\}\,, \tag{4.7}
$$

we can predict exactly the $1/(d-4)^4$ and $1/(d-4)^3$ poles of the C_F^2 part of the two-loop form factor $G_1^{(2l)}$ (eqs. (5.3), (5.4) below). Moreover, exponentiating also the finite part of $G_1^{(1l)}$, the double pole of eq. (5.5) is exactly recovered.

5. Two-loop form factors

In this section we collect the analytic expression of the two-loop renormalized form factors defined in eq. (2.5). In the expressions below, C_A is the Casimir operator of the adjoint representation of SU(N_c), $C_A = N_c$, T_R is the normalization factor of the color matrices, $T_R = 1/2$, N_l is the number of massless quarks in the theory, and N_h is the number of quarks of mass m_b. Therefore, for the decay $b \to u\, W^*$ in the SM, $N_l = 4$, and $N_h = 1$. In the finite part of the form factors given below, the constant $\mathcal{K}$ is a rational number. Its numerical value is $\mathcal{K} = 3.32812 \pm 0.00002$, and its analytical value is likely to be $\mathcal{K} = 213/64$. We observe that the following formulas involve HPLs of argument x and of maximal weight 4. If desired, the HPLs appearing in the equations below can all be rewritten in terms of product of Nielsen Polylogarithms of more complicated argument. Because of the chosen renormalization scheme, our results depend on the renormalization scale μ. In the formulas below, we employ the following notation:

$$
\ln \left(\frac{\mu^2}{m_b^2} \right) \equiv L_\mu\,. \tag{5.1}
$$

The form factor $G_1^{(2l)}$ can be written as

$$
G_1^{(2l)} = C^2(d) \left(\frac{\mu^2}{m^2} \right)^{4-d} C_F \sum_{i=-4}^{0} G_1^{(2l,i)} (d-4)^i + \mathcal{O}\left(d-4 \right) \,, \tag{5.2}
$$

where the coefficient of the expansion in $(d-4)$ (up to the finite term) are

$$G_1^{(2l,-4)} = C_F\frac{1}{2}\,, \tag{5.3}$$

$$G_1^{(2l,-3)} = C_F\left[-\frac{5}{4}-H(1;x)\right]-C_A\frac{11}{8}+T_RN_l\frac{1}{2}\,, \tag{5.4}$$

$$\begin{aligned}
G_1^{(2l,-2)} &= C_F\left[\frac{73}{32}-\frac{1-8x}{4x}H(1;x)+\frac{1}{2}H(0,1;x)+2H(1,1;x)\right]+C_A\left[\frac{49-66L_\mu+9\zeta(2)}{72}\right.\\
&\quad\left.+\frac{11}{12}H(1;x)\right]+T_RN_l\left[\frac{-5+6L_\mu}{18}-\frac{1}{3}H(1;x)\right]+T_RN_h\left[\frac{1}{3}L_\mu\right],
\end{aligned} \tag{5.5}$$

$$\begin{aligned}
G_1^{(2l,-1)} &= C_F\left[-\frac{3(71+8\zeta(2)-16\zeta(3))}{64}+\frac{13-55x}{16x}H(1;x)+\frac{1-8x}{8x}H(0,1;x)\right.\\
&\quad\left.+\frac{3-14x}{4x}H(1,1;x)-\frac{1}{4}H(0,0,1;x)-\frac{3}{2}H(0,1,1;x)-H(1,0,1;x)-4H(1,1,1;x)\right]\\
&\quad+C_A\left[\frac{1549+1980L_\mu-396L_\mu^2+972\zeta(2)-1188\zeta(3)}{1728}+\frac{67+66L_\mu-18\zeta(2)}{72}H(1;x)\right]\\
&\quad+T_RN_l\left[\frac{-125-180L_\mu+36L_\mu^2-108\zeta(2)}{432}-\frac{5+6L_\mu}{18}H(1;x)\right]\\
&\quad+T_RN_h\left[\frac{-5L_\mu+L_\mu^2-\zeta(2)}{12}-\frac{L_\mu}{3}H(1;x)\right],\\
G_1^{(2l,0)} &= C_F\left[\frac{1}{1280(x-1)^3}(6635(x-1)^3+(80(x-1)(-59-62x+3x^2)-480\ln(2)(10\right.\\
&\quad-22x+5x^2+4x^3))\zeta(2)-16(261-64\mathcal{K}+41x+192\mathcal{K}x+405x^2-192\mathcal{K}x^2\\
&\quad-99x^3+64\mathcal{K}x^3)\zeta^2(2)+40(52-72x+15x^2+14x^3)\zeta(3))+\frac{1}{4(x-1)^2x}(\zeta(2)\\
&\quad-3x\zeta(2)+x^2\zeta(2)+5x^3\zeta(2))H(-1;x)+\frac{1}{32(x-1)^2x}(-49+251x-355x^2\\
&\quad+153x^3+12\zeta(2)-160x\zeta(2)-148x^2\zeta(2)+24x^3\zeta(2)-16x\zeta(3)+32x^2\zeta(3)\\
&\quad-16x^3\zeta(3))H(1;x)+\frac{3(-2\zeta(2)-2x\zeta(2)+x^2\zeta(2))}{8(x-1)^3}H(2;x)\\
&\quad+\frac{\zeta(2)-5x\zeta(2)+3x^2\zeta(2)-x^3\zeta(2)}{2(x-1)^3}H(0,-1;x)+\frac{1}{32(x-1)^3x}(15-106x\\
&\quad+248x^2-138x^3-19x^4+72x\zeta(2)+152x^2\zeta(2)+48x^3\zeta(2))H(0,1;x)\\
&\quad+\frac{25-134x+59x^2}{16(x-1)x}H(1,1;x)+\frac{1-3x+x^2+5x^3}{2(x-1)^2x}H(-1,0,1;x)\\
&\quad-\frac{1+27x-9x^2-25x^3+12x^4}{16(x-1)^3x}H(0,0,1;x)+\frac{5-59x+83x^2-56x^3+30x^4}{8(x-1)^3x}H(0,1,1;x)\\
&\quad-\frac{1-x+21x^2-7x^3}{4(x-1)^2x}H(1,0,1;x)-\frac{7-26x}{4x}H(1,1,1;x)-\frac{2+2x-x^2}{8(x-1)^3}H(2,1,1;x)\\
&\quad+\frac{1-5x+3x^2-x^3}{(x-1)^3}H(0,-1,0,1;x)-\frac{3+11x-5x^2+3x^3}{8(x-1)^3}H(0,0,0,1;x)
\end{aligned}$$

$$-\frac{9+9x+13x^2-3x^3}{4(x-1)^3}H(0,0,1,1;x)+\frac{18x-7x^2+3x^3}{4(x-1)^3}H(0,1,0,1;x)+3H(1,0,1,1;x)$$
$$+\frac{7}{2}H(0,1,1,1;x)-\frac{1}{2}H(1,0,0,1;x)+2H(1,1,0,1;x)+8H(1,1,1,1;x)\Big]$$
$$+C_A\Big[\frac{29700L_\mu^2-447185-142560L_\mu-3960L_\mu^3}{103680}+\frac{1}{103680(x-1)^3}((11880L_\mu(x-1)^3$$
$$-180(x-1)(517+982x+913x^2)+19440\ln(2)(10-22x+5x^2+4x^3))\zeta(2)$$
$$+648(71-64\mathcal{K}-631x+192\mathcal{K}x+159x^2-192\mathcal{K}x^2-125x^3+64\mathcal{K}x^3)\zeta^2(2)$$
$$+180(-698+1338x-825x^2+104x^3)\zeta(3))-\frac{(1-3x+x^2+5x^3)\zeta(2)}{8(x-1)^2x}H(-1;x)$$
$$+\frac{1}{864(x-1)^2x}(807+198L_\mu-4159x-990L_\mu x+198L_\mu^2x+5897x^2+1386L_\mu x^2$$
$$-396L_\mu^2x^2-2545x^3-594L_\mu x^3+198L_\mu^2x^3-(216+1368x+342x^2+1206x^3)\zeta(2)$$
$$+(756x-1512x^2+756x^3)\zeta(3))H(1;x)+\frac{3(2+2x-x^2)\zeta(2)}{16(x-1)^3}H(2;x)$$
$$-\frac{(1-5x+3x^2-x^3)\zeta(2)}{4(x-1)^3}H(0,-1;x)+\frac{1}{144(x-1)^3x}(-33+260x+66L_\mu x$$
$$-384x^2-198L_\mu x^2+273x^3+198L_\mu x^3-116x^4-66L_\mu x^4-18x\zeta(2)+468x^2\zeta(2)$$
$$+72x^4\zeta(2))H(0,1;x)+\frac{1}{144(x-1)x}(-39+235x+132L_\mu x-349x^2-132L_\mu x^2$$
$$-72x\zeta(2)+72x^2\zeta(2))H(1,1;x)-\frac{1-3x+x^2+5x^3}{4(x-1)^2x}H(-1,0,1;x)$$
$$-\frac{8+36x-33x^2-20x^3}{48(x-1)^3}H(0,0,1;x)+\frac{2-168x+219x^2-62x^3}{48(x-1)^3}H(0,1,1;x)$$
$$-\frac{47-49x+44x^2}{48(x-1)^2}H(1,0,1;x)-\frac{11}{6}H(1,1,1;x)+\frac{2+2x-x^2}{16(x-1)^3}H(2,1,1;x)$$
$$-\frac{1-5x+3x^2-x^3}{2(x-1)^3}H(0,-1,0,1;x)-\frac{1+10x+4x^2}{8(x-1)^3}H(0,0,0,1;x)$$
$$-\frac{1+2x+4x^2}{4(x-1)^3}H(0,0,1,1;x)+\frac{1+2x+4x^2}{8(x-1)^3}H(0,1,0,1;x)+\frac{1}{2}H(1,0,0,1;x)\Big]$$
$$+T_RN_l\Big[\frac{1}{5184}(6629+2592L_\mu-540L_\mu^2+72L_\mu^3+3420\zeta(2)-216L_\mu\zeta(2)+720\zeta(3))$$
$$-\frac{57+18L_\mu-209x-54L_\mu x+18L_\mu^2x-90x\zeta(2)}{216x}H(1;x)-\frac{3-19x-6L_\mu x}{36x}H(0,1;x)$$
$$-\frac{3-19x-6L_\mu x}{18x}H(1,1;x)+\frac{1}{6}H(0,0,1;x)+\frac{1}{3}H(0,1,1;x)+\frac{1}{3}H(1,0,1;x)$$
$$+\frac{2}{3}H(1,1,1;x)\Big]+T_RN_h\Big[\frac{1}{2592(x-1)^3}(-1111-1296L_\mu+270L_\mu^2-36L_\mu^3$$
$$+7869x+3888L_\mu x-810L_\mu^2x+108L_\mu^3x-14709x^2-3888L_\mu x^2+810L_\mu^2x^2$$

$$-108L_\mu^3x^2+7951x^3+1296L_\mu x^3-270L_\mu^2x^3+36L_\mu^3x^3-414\zeta(2)+108L_\mu\zeta(2)$$
$$-5670x\zeta(2)-324L_\mu x\zeta(2)+9126x^2\zeta(2)+324L_\mu x^2\zeta(2)-738x^3\zeta(2)$$
$$-108L_\mu x^3\zeta(2)+504\zeta(3)+1080x\zeta(3)+1512x^2\zeta(3)-504x^3\zeta(3))$$
$$+\frac{1}{216(x-1)^2x}(-57-18L_\mu-89x+90L_\mu x-18L_\mu^2x+73x^2$$
$$-126L_\mu x^2+36L_\mu^2x^2+265x^3+54L_\mu x^3-18L_\mu^2x^3+18x\zeta(2)-36x^2\zeta(2)$$
$$+18x^3\zeta(2))H(1;x)+\frac{3+8x-24x^3-19x^4-(6x-18x^2+18x^3-6x^4)L_\mu}{36(x-1)^3x}H(0,1;x)$$
$$+\frac{1}{3}L_\mu H(1,1;x)-\frac{1+3x+3x^2-x^3}{6(x-1)^3}H(0,0,1;x)\Bigg]. \tag{5.6}$$

The form factor $G_2^{(2l)}$ is

$$G_2^{(2l)} = C^2(d)\left(\frac{\mu^2}{m^2}\right)^{4-d}C_F\sum_{i=-2}^{0}G_2^{(2l,i)}(d-4)^i+\mathcal{O}(d-4)\,, \tag{5.7}$$

where

$$G_2^{(2l,-2)} = C_F\left[-\frac{1}{x}+\frac{2-3x}{2x^2}H(1;x)\right], \tag{5.8}$$
$$G_2^{(2l,-1)} = C_F\left[\frac{9}{4x}-\frac{7(2-5x)}{8x^2}H(1;x)-\frac{(2-3x)}{4x^2}H(0,1;x)-\frac{3(2-3x)}{2x^2}H(1,1;x)\right], \tag{5.9}$$
$$G_2^{(2l,0)} = C_F\Bigg[\frac{1}{80(x-1)^4x}(-310(x-1)^4+(60\ln(2)x(-38+58x-40x^2+11x^3)$$
$$-20(x-1)(10-120x-79x^2+12x^3))\zeta(2)+16x(125+103x)\zeta^2(2)$$
$$-5x(-30+110x-80x^2+27x^3)\zeta(3))+\frac{(2-9x-5x^2+3x^3-3x^4)\zeta(2)}{2(x-1)^3x^2}H(-1;x)$$
$$+\frac{1}{16(x-1)^3x^2}(-32+195x-397x^2+337x^3-103x^4+24\zeta(2)-12x\zeta(2)$$
$$+844x^2\zeta(2)-76x^3\zeta(2)+36x^4\zeta(2))H(1;x)+\frac{3(30-34x+16x^2-3x^3)\zeta(2)}{4(x-1)^4}H(2;x)$$
$$+\frac{2(2+x)\zeta(2)}{(x-1)^4}H(0,-1;x)+\frac{1}{16(x-1)^4x^2}(26-69x-68x^2-58x^3+166x^4$$
$$+3x^5-(448x^2+368x^3)\zeta(2))H(0,1;x)+\frac{8-18x+37x^2+172x^3-49x^4}{8(x-1)^2x^3}H(1,1;x)$$
$$+\frac{2-9x-5x^2+3x^3-3x^4}{(x-1)^3x^2}H(-1,0,1;x)+\frac{30-34x+16x^2-3x^3}{4(x-1)^4}H(2,1,1;x)$$
$$-\frac{2-27x+4x^2-48x^3+66x^4-15x^5}{8(x-1)^4x^2}H(0,0,1;x)+\frac{7(2-3x)}{2x^2}H(1,1,1;x)$$
$$+\frac{10-39x+234x^2-276x^3+86x^4-24x^5}{4(x-1)^4x^2}H(0,1,1;x)+\frac{4(2+x)}{(x-1)^4}H(0,-1,0,1;x)$$

$$
\begin{aligned}
&-\frac{2-13x-22x^2-12x^3+3x^4}{2(x-1)^3x^2}H(1,0,1;x)+\frac{(4+5x)}{(x-1)^4}H(0,0,0,1;x)\\
&+\frac{6(4+3x)}{(x-1)^4}H(0,0,1,1;x)-\frac{3(4+3x)}{(x-1)^4}H(0,1,0,1;x)\Bigg]\\
&+C_A\Bigg[\frac{1}{1440(x-1)^4x}(1320L_\mu(x-1)^4+20(x-1)^3(-269+242x)\\
&+(540(x-1)(4+50x+5x^2+8x^3)+540\ln(2)x(38-58x+40x^2-11x^3))\zeta(2)\\
&+36x(364+317x+108x^2)\zeta^2(2)+45x(-30+110x-80x^2+27x^3)\zeta(3))\\
&-\frac{(2-9x-5x^2+3x^3-3x^4)\zeta(2)}{4(x-1)^3x^2}H(-1;x)+\frac{1}{144(x-1)^3x^2}(406+132L_\mu-2067x\\
&-594L_\mu x+3603x^2+990L_\mu x^2-2629x^3-726L_\mu x^3+687x^4+198L_\mu x^4-(144-864x\\
&-1224x^2-1242x^3+54x^4)\zeta(2))H(1;x)-\frac{3(30-34x+16x^2-3x^3)\zeta(2)}{8(x-1)^4}H(2;x)\\
&-\frac{(2+x)\zeta(2)}{(x-1)^4}H(0,-1;x)+\frac{1}{24(x-1)^4x^2}(-22+145x-300x^2+109x^3+59x^4\\
&+9x^5-(240x^2+210x^3+72x^4)\zeta(2))H(0,1;x)-\frac{26-151x+14x^2-42x^3}{24(x-1)^2x^2}H(1,1;x)\\
&-\frac{2-9x-5x^2+3x^3-3x^4}{2(x-1)^3x^2}H(-1,0,1;x)+\frac{4+76x-96x^2+16x^3-9x^4}{8(x-1)^4x}H(0,0,1;x)\\
&+\frac{8+58x-24x^2-36x^3+3x^4}{8(x-1)^4x}H(0,1,1;x)+\frac{4+24x+11x^2+3x^3}{8(x-1)^3x}H(1,0,1;x)\\
&-\frac{30-34x+16x^2-3x^3}{8(x-1)^4}H(2,1,1;x)+\frac{24+17x+4x^2}{4(x-1)^4}H(0,0,0,1;x)\\
&-\frac{2(2+x)}{(x-1)^4}H(0,-1,0,1;x)+\frac{8+9x+4x^2}{4(x-1)^4}(2H(0,0,1,1;x)-H(0,1,0,1;x))\Bigg]\\
&+T_RN_l\Bigg[-\frac{19+6L_\mu}{18x}+\frac{26-51x+6(2-3x)L_\mu}{36x^2}H(1;x)+\frac{2-3x}{6x^2}(H(0,1;x)+2H(1,1;x))\Bigg]\\
&+T_RN_h\Bigg[\frac{1}{18(x-1)^4x}(-19-6L_\mu-164x+24L_\mu x+393x^2-36L_\mu x^2-218x^3\\
&+24L_\mu x^3+8x^4-6L_\mu x^4+(252x-300x^2+84x^3-36x^4)\zeta(2)-(72x+36x^2)\zeta(3))\\
&+\frac{26-223x-124x^2-51x^3+(12-42x+48x^2-18x^3)L_\mu}{36(x-1)^2x^2}H(1;x)\\
&-\frac{2-9x-21x^2-13x^3-3x^4}{6(x-1)^3x^2}H(0,1;x)+\frac{2(2+x)}{(x-1)^4}H(0,0,1;x)\Bigg]. \qquad (5.10)
\end{aligned}
$$

The form factor $G_3^{(2l)}$ can be written as

$$
G_3^{(2l)} = C^2(d)\left(\frac{\mu^2}{m^2}\right)^{4-d} C_F\sum_{i=-2}^{0} G_3^{(2l,i)}(d-4)^i + \mathcal{O}(d-4)\,, \qquad (5.11)
$$

with

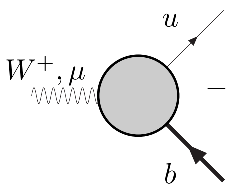
$$
\begin{aligned}
G_3^{(2l,-2)} &= C_F\left[\frac{1}{2x}H(1;x)\right], && (5.12)\\
G_3^{(2l,-1)} &= C_F\left[-\frac{13}{8x}H(1;x)-\frac{1}{4x}(H(0,1;x)+6H(1,1;x))\right], && (5.13)\\
G_3^{(2l,0)} &= C_F\left[\frac{1}{80(x-1)^4}((-60(x-1)x(57+2x)-60\ln(2)(-22+30x-26x^2+9x^3))\zeta(2)\right.\\
&\quad -48(9+49x+18x^2)\zeta^2(2)-5(22-90x+82x^2-41x^3)\zeta(3))-\frac{1}{16(x-1)^3x}(51-157x\\
&\quad +161x^2-55x^3-(12-364x-436x^2-28x^3)\zeta(2))H(1;x)\\
&\quad +\frac{(1+5x+5x^2+x^3)\zeta(2)}{2(x-1)^3x}H(-1;x)+\frac{3(-14+6x-2x^2+x^3)\zeta(2)}{4(x-1)^4}H(2;x)\\
&\quad -\frac{6x\zeta(2)}{(x-1)^4}H(0,-1;x)+\frac{1}{16(x-1)^4x}(11-72x+366x^2-260x^3-45x^4+96x\zeta(2)\\
&\quad +528x^2\zeta(2)+192x^3\zeta(2))H(0,1;x)-\frac{1+36x+8x^2-24x^3-3x^4}{8(x-1)^4x}H(0,0,1;x)\\
&\quad +\frac{25-174x-x^2}{8(x-1)^2x}H(1,1;x)+\frac{1+5x+5x^2+x^3}{(x-1)^3x}H(-1,0,1;x)+\frac{7}{(2x)}H(1,1,1;x)\\
&\quad +\frac{5-78x+4x^2+70x^3+8x^4}{4(x-1)^4x}H(0,1,1;x)-\frac{1+12x+32x^2-3x^3}{2(x-1)^3x}H(1,0,1;x)\\
&\quad -\frac{14-6x+2x^2-x^3}{4(x-1)^4}H(2,1,1;x)-\frac{2(2+15x+4x^2)}{(x-1)^4}H(0,0,1,1;x)\\
&\quad \left.-\frac{2+3x+4x^2}{(x-1)^4}H(0,0,0,1;x)-\frac{12x}{(x-1)^4}H(0,-1,0,1;x)+\frac{2+15x+4x^2}{(x-1)^4}H(0,1,0,1;x)\right]\\
&\quad +C_A\left[\frac{1}{160(x-1)^4}(60(x-1)^3+(-20(x-1)(56+105x+40x^2)+60\ln(2)(-22\right.\\
&\quad +30x-26x^2+9x^3))\zeta(2)-12(18+137x+108x^2)\zeta^2(2)-5(-22+90x-82x^2\\
&\quad +41x^3)\zeta(3))-\frac{(1+5x+5x^2+x^3)\zeta(2)}{4(x-1)^3x}H(-1;x)+\frac{1}{144(x-1)^3x}(335+66L_\mu\\
&\quad -843x-198L_\mu x+681x^2+198L_\mu x^2-173x^3-66L_\mu x^3-(72+468x+2466x^2\\
&\quad +126x^3)\zeta(2))H(1;x)-\frac{3(-14\zeta(2)+6x\zeta(2)-2x^2\zeta(2)+x^3\zeta(2))}{8(x-1)^4}H(2;x)\\
&\quad +\frac{3x\zeta(2)}{(x-1)^4}H(0,-1;x)-\frac{1}{24(x-1)^4x}(11+22x-171x^2+79x^3+59x^4-(36x\\
&\quad +270x^2+216x^3)\zeta(2))H(0,1;x)-\frac{20+40x-66x^2-3x^3}{8(x-1)^4}H(0,0,1;x)\\
&\quad -\frac{1+5x+5x^2+x^3}{2(x-1)^3x}H(-1,0,1;x)-\frac{13+82x+58x^2}{24(x-1)^2x}H(1,1;x)\\
&\quad -\frac{22+56x-62x^2-7x^3}{8(x-1)^4}H(0,1,1;x)-\frac{10+23x+9x^2}{8(x-1)^3}H(1,0,1;x)
\end{aligned}
$$

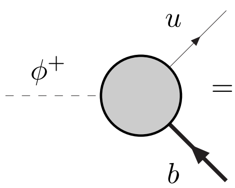
$$
\begin{aligned}
&+\frac{14-6x+2x^2-x^3}{8(x-1)^4}H(2,1,1;x)-\frac{2+31x+12x^2}{4(x-1)^4}H(0,0,0,1;x)\\
&+\frac{6x}{(x-1)^4}H(0,-1,0,1;x)-\frac{2+7x+12x^2}{4(x-1)^4}(2H(0,0,1,1;x)-H(0,1,0,1;x))\Bigg]\\
&+T_RN_l\left[\frac{(25+6L_\mu)}{36x}H(1;x)+\frac{1}{6x}(H(0,1;x)+2H(1,1;x))\right]\\
&+T_RN_h\Bigg[\frac{(54-91x+20x^2+17x^3-(36+12x-52x^2+4x^3)\zeta(2)+36x\zeta(3))}{6(x-1)^4}\\
&+\frac{25+322x+25x^2+(6-12x+6x^2)L_\mu}{36(x-1)^2x}H(1;x)-\frac{1+21x+21x^2+x^3}{6(x-1)^3x}H(0,1;x)\\
&-\frac{6x}{(x-1)^4}H(0,0,1;x)\Bigg]\,. \qquad (5.14)
\end{aligned}
$$

We checked our results for the form factors $G_i^{(2l)}(x)$ $i=1,2,3$ against the calculation of Martin Beneke, Tobias Huber, and Xin-Quing Li [37] and we found complete analytical agreement.

6. Ward identities

We explicitly checked that the UV renormalized form factors satisfy the on-shell Ward identity[2]

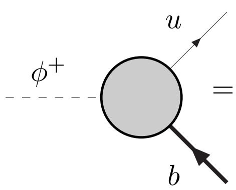
iq_μ 

$- M_W$ 

$= 0\,,$ (6.1)

where ϕ is the charged pseudo-Goldstone boson and the gray circles represents the sum of all two-loop one-particle-irreducible QCD corrections to the vertices. The Lorentz index associated to the W-boson is saturated by the boson momentum q^μ. In order to satisfy the relation in eq. (6.1) it is necessary to renormalize also the factor m_b appearing in the tree-level ϕ^+ub coupling. The relevant NNLO mass counter term can be found in [36].

The two-loop corrections to the scalar coupling of the pseudo-Goldstone boson to quark can be absorbed in a single form-factor S, defined as follows



$= -\dfrac{m_b}{M_W}\; S(q^2)\,\overline{u}(p)\,(1-\gamma_5)\,u(P)\,.$ (6.2)

[2]It can be proved that the Ward identity is fulfilled already at the level of master integrals, irrespectively on the analytic expression of the MIs themselves.

The UV renormalized form factor has the following perturbative expansion in α_s:

$$S = \frac{ig_w}{2\sqrt{2}} V_{ub} \left[S^{(0l)} + \left(\frac{\alpha_s}{\pi}\right) S^{(1l)} + \left(\frac{\alpha_s}{\pi}\right)^2 S^{(2l)} + \mathcal{O}\left(\frac{\alpha_s^3}{\pi^3}\right)\right] , \tag{6.3}$$

with $S^{(0l)} = 1$.

The one-loop form factor $S^{(1l)}$ is given by

$$S^{(1l)} = C(d) \left(\frac{\mu^2}{m^2}\right)^{(4-d)/2} C_F \sum_{i=-2}^{1} S^{(1l,i)}(d-4)^i + \mathcal{O}\left((d-4)^2\right) . \tag{6.4}$$

After UV renormalization (including the renormalization of the Yukawa $\phi^+ ub$ coupling), the coefficients of the expansion in $(d-4)$ are

$$S^{(1l,-2)} = -1 \, , \tag{6.5}$$

$$S^{(1l,-1)} = \frac{5}{4} + H(1;x) \, , \tag{6.6}$$

$$S^{(1l,0)} = -1 - \frac{1}{2x} H(1;x) - H(1,1;x) - \frac{1}{2} H(0,1;x) \, , \tag{6.7}$$

$$\begin{aligned} S^{(1l,1)} = {} & 1 + \frac{x+1}{4x} H(1;x) + \frac{1}{2x} H(1,1;x) + \frac{1}{4x} H(0,1;x) + H(1,1,1;x) \\ & + \frac{1}{2} H(1,0,1;x) + \frac{1}{2} H(0,1,1;x) + \frac{1}{4} H(0,0,1;x). \end{aligned} \tag{6.8}$$

The two-loop form factor $S^{(2l)}$ is given by

$$S^{(2l)} = C^2(d) \left(\frac{\mu^2}{m^2}\right)^{4-d} C_F \sum_{i=-4}^{0} S^{(2l,i)}(d-4)^i + \mathcal{O}\left(d-4\right) . \tag{6.9}$$

where the coefficient of the expansion in $(d-4)$ are

$$S^{(2l,-4)} = C_F \frac{1}{2} \, , \tag{6.10}$$

$$S^{(2l,-3)} = C_F\left[-\frac{5}{4} - H(1;x)\right] - C_A \frac{11}{8} + T_R N_l \frac{1}{2} \, , \tag{6.11}$$

$$\begin{aligned} S^{(2l,-2)} = {} & C_F\left[\frac{57}{32} + \frac{2+5x}{(4x)} H(1;x) + \frac{1}{2} H(0,1;x) + 2H(1,1;x)\right] + C_A\left[\frac{49+9\zeta(2)}{72}\right. \\ & \left. - \frac{11}{12} L_\mu + \frac{11}{12} H(1;x)\right] + T_R N_l\left[-\frac{5}{18} + \frac{1}{3} L_\mu - \frac{1}{3} H(1;x)\right] + T_R N_h\left[\frac{1}{3} L_\mu\right] , \end{aligned} \tag{6.12}$$

$$\begin{aligned} S^{(2l,-1)} = {} & C_F\left[-\frac{3(47+8\zeta(2)-16\zeta(3))}{64} - \frac{7+10x}{8x} H(1;x) - \frac{2+5x}{8x} H(0,1;x)\right. \\ & \left. - \frac{6+5x}{4x} H(1,1;x) - \frac{1}{4} H(0,0,1;x) - \frac{3}{2} H(0,1,1;x) - H(1,0,1;x) - 4H(1,1,1;x)\right] \\ & + C_A\left[\frac{(1549+1980L_\mu - 396L_\mu^2 + 972\zeta(2) - 1188\zeta(3))}{1728} + \frac{67+66L_\mu - 18\zeta(2)}{72} H(1;x)\right] \\ & + T_R N_l\left[\frac{-125-180L_\mu + 36L_\mu^2 - 108\zeta(2)}{432} - \frac{5+6L_\mu}{18} H(1;x)\right] \\ & + T_R N_h\left[\frac{-5L_\mu + L_\mu^2 - \zeta(2)}{12} - \frac{1}{3} L_\mu H(1;x)\right] , \end{aligned}$$

$$
\begin{aligned}
S^{(2l,0)} = C_F\Bigg[&\frac{831}{256}+\left(\frac{3(13-7x)}{16(x-1)}+\frac{3\ln(2)(x-4)(7x-8)}{8(x-1)^2}\right)\zeta(2)+\left(\frac{9(5-34x+11x^2)}{80(x-1)^2}\right.\\
&\left.-\frac{4K}{5}\right)\zeta^2(2)-\frac{(74-96x+13x^2)\zeta(3)}{32(x-1)^2}+\frac{(1+2x+x^2)\zeta(2)}{2(x-1)x}H(-1;x)\\
&-\frac{17+8x-25x^2-(12-78x+30x^2)\zeta(2)-(8x-8x^2)\zeta(3)}{16(x-1)x}H(1;x)\\
&-\frac{9(4-4x+x^2)\zeta(2)}{8(x-1)^2}H(2;x)-\frac{\zeta(2)}{2}H(0,-1;x)+\frac{1}{16(x-1)^2x}(11+12x-15x^2\\
&-8x^3+12x\zeta(2)+24x^2\zeta(2))H(0,1;x)+\frac{4-x+5x^2}{8x^2}H(1,1;x)\\
&+\frac{1+2x+x^2}{(x-1)x}H(-1,0,1;x)-\frac{2-13x+20x^2-3x^3}{16(x-1)^2x}H(0,0,1;x)\\
&+\frac{14+5x}{4x}H(1,1,1;x)+\frac{10-33x+20x^2+6x^3}{8(x-1)^2x}H(0,1,1;x)\\
&-\frac{2+5x-4x^2}{4(x-1)x}H(1,0,1;x)-\frac{3(4-4x+x^2)}{8(x-1)^2}H(2,1,1;x)-H(0,-1,0,1;x)\\
&-\frac{5-2x+3x^2}{8(x-1)^2}H(0,0,0,1;x)+\frac{1-10x+3x^2}{4(x-1)^2}H(0,0,1,1;x)+\frac{7}{2}H(0,1,1,1;x)\\
&+\frac{4-4x+3x^2}{4(x-1)^2}H(0,1,0,1;x)-\frac{1}{2}H(1,0,0,1;x)+3H(1,0,1,1;x)\\
&+2H(1,1,0,1;x)+8H(1,1,1,1;x)\Bigg]+C_A\Bigg[-\frac{54589}{20736}-\frac{11L_\mu}{12}+\frac{55L_\mu^2}{192}-\frac{11L_\mu^3}{288}\\
&+\left(\frac{11L_\mu}{96}-\frac{3\ln(2)(x-4)(7x-8)}{16(x-1)^2}-\frac{1067+49x}{576(x-1)}\right)\zeta(2)-\left(\frac{179-250x+125x^2}{160(x-1)^2}\right.\\
&\left.-\frac{2K}{5}\right)\zeta^2(2)+\frac{(896-1324x+347x^2)\zeta(3)}{576(x-1)^2}-\frac{(1+2x+x^2)\zeta(2)}{4(x-1)x}H(-1;x)\\
&+\frac{1}{432(x-1)x}(708+198L_\mu-466x-198L_\mu x-99L_\mu^2x-242x^2+99L_\mu^2x^2\\
&-216\zeta(2)+738x\zeta(2)-684x^2\zeta(2)-378x\zeta(3)+378x^2\zeta(3))H(1;x)\\
&+\frac{9(4-4x+x^2)\zeta(2)}{(16(x-1)^2)}H(2;x)+\frac{1}{4}\zeta(2)H(0,-1;x)-\frac{1}{144(x-1)^2x}(66+56x\\
&+66L_\mu x-211x^2-132L_\mu x^2+89x^3+66L_\mu x^3-(126x-144x^2+72x^3)\zeta(2))H(0,1;x)\\
&-\frac{78+223x+132L_\mu x-72x\zeta(2)}{144x}H(1,1;x)-\frac{1+2x+x^2}{2(x-1)x}H(-1,0,1;x)\\
&-\frac{40-56x+7x^2}{48(x-1)^2}H(0,0,1;x)-\frac{11}{6}H(1,1,1;x)-\frac{44-88x+53x^2}{48(x-1)^2}H(0,1,1;x)\\
&+\frac{29-35x}{48(x-1)}H(1,0,1;x)+\frac{3(4-4x+x^2)}{16(x-1)^2}H(2,1,1;x)+\frac{1}{2}H(0,-1,0,1;x)\\
&-\frac{1}{8(x-1)^2}H(0,0,0,1;x)-\frac{1}{4(x-1)^2}H(0,0,1,1;x)+\frac{1}{8(x-1)^2}H(0,1,0,1;x)
\end{aligned}
$$

$$+\frac{1}{2}H(1,0,0,1;x)\Big]+T_RN_l\Big[\frac{1}{5184}(3893+1728L_\mu-540L_\mu^2+72L_\mu^3+3420\zeta(2)$$
$$-216L_\mu\zeta(2)+720\zeta(3))+\frac{48+18L_\mu+28x-9L_\mu^2x+45x\zeta(2)}{108x}H(1;x)$$
$$+\frac{3+5x+3L_\mu x}{18x}H(0,1;x)+\frac{3+5x+3L_\mu x}{9x}H(1,1;x)+\frac{1}{6}H(0,0,1;x)$$
$$+\frac{1}{3}H(0,1,1;x)+\frac{1}{3}H(1,0,1;x)+\frac{2}{3}H(1,1,1;x)\Big]+T_RN_h\Big[\frac{L_\mu}{3}-\frac{5L_\mu^2}{48}+\frac{L_\mu^3}{72}$$
$$+\frac{11407-17630x+8527x^2}{2592(x-1)^2}-\left(\frac{L_\mu}{24}-\frac{409-747x+651x^2-185x^3}{144(x-1)^3}\right)\zeta(2)$$
$$-\frac{7\zeta(3)}{36}+\frac{1}{108(x-1)^2x}(48+18L_\mu+104x-36L_\mu x-9L_\mu^2x-112x^2+18L_\mu x^2$$
$$+18L_\mu^2x^2+56x^3-9L_\mu^2x^3+9x\zeta(2)-18x^2\zeta(2)+9x^3\zeta(2))H(1;x)$$
$$-\frac{1}{18(x-1)^3x}(3+14x+3L_\mu x-9L_\mu x^2-6x^3+9L_\mu x^3+5x^4-3L_\mu x^4)H(0,1;x)$$
$$+\frac{L_\mu}{3}H(1,1;x)+\frac{1}{6}H(0,0,1;x)\Big]. \qquad (6.13)$$

When written in terms of form factors, the Ward identity in eq. (6.1) reads as follows:

$$2G_1^{(2l)}(x)+xG_2^{(2l)}(x)+G_3^{(2l)}(x)-2S^{(2l)}(x)=0\,. \qquad (6.14)$$

It can be checked that the form factors presented in this paper fulfill eq. (6.14).

7. Conclusions

In this paper, we presented analytic expressions for the two-loop QCD corrections to the decay process $b\to u\,W^*\to u\,l\,\overline{\nu}$. This process is important for the precise determination of the CKM matrix element V_{ub} and, therefore, for the study of flavor and CP violation within and beyond the Standard Model of fundamental interactions.

The Lorentz structure of the process is parametrized in terms of three form factors, whose analytic expression are given in the form of a Laurent series of $(d-4)$, where d is the space-time dimension. The coefficients of the series are expressed in the well known functional basis of harmonic polylogarithms of a single dimensionless variable. The result can be used in a SCET framework, after combining it with the jet and soft functions already known in the literature, for a phenomenological determination of $|V_{ub}|$. The results presented here are the first step towards a complete determination of the NNLO QCD corrections to the heavy-to-light quark transition.

Acknowledgments

We are grateful to U. Aglietti, for proposing the subject of the paper, and to M. Beneke for allowing us to compare our results with the ones obtained by his group. We thank U. Aglietti and P. Gambino for carefully reading the manuscript and providing us with

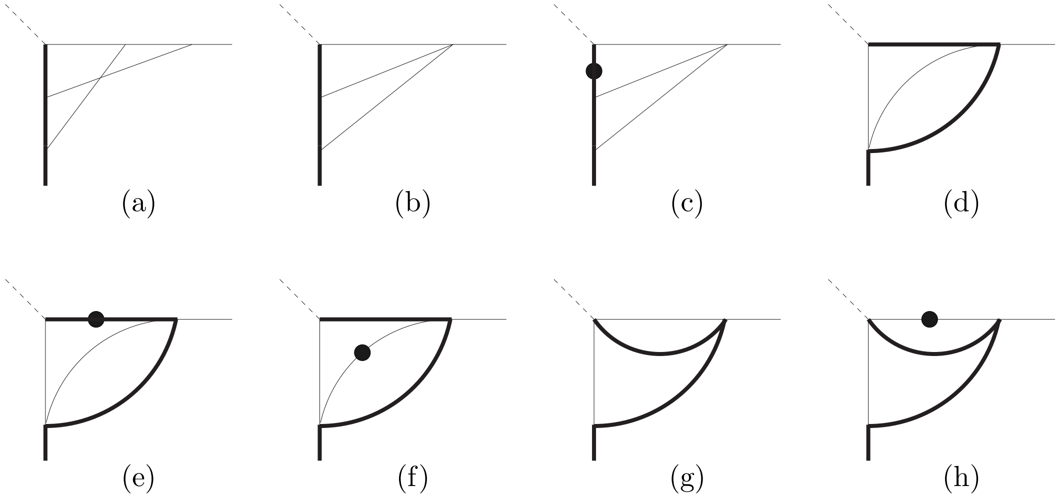


Figure 3: Master Integrals needed for the Two-loop QCD corrections. Thick lines represent massive particles, thin lines represent massless ones.

valuable feedback. We wish to thank T. Becher, C. Greub, T. Gehrmann, and B. Pecjak for useful discussions. We are indebted with R. Boughezal and G. Bell for several numerical and analytical cross-checks of the master integrals. We are grateful to J. Vermaseren for his kind assistance in the use of `FORM` [38], and to the authors of the packages `AIR` [39] and `FIESTA` [40], that were employed in partial checks of the calculation.

R.B. wishes to thank the Theoretical Physics Department of the University of Florence for kind hospitality during a part of this work.

This work was supported by the Swiss National Science Foundation (SNF) under contract 200020-117602.

Note added. While this paper was with the editors, the results were independently confirmed by three different groups [37, 45, 46].

A. The master integrals

In this appendix we collect the analytic expressions of the Master Integrals for the Feynman diagrams of figure 3. We provide only eight of them, since the other MIs can be found in [41, 42]. It must be pointed out that the MIs (a)–(f) in figure 3 were already calculated in [43]. We checked the analytic expressions that we obtained against the results in [43] and we found complete agreement. Moreover, all the MIs were checked by comparing their numerical value to the results obtained by direct numerical integration with the sector decomposition method. The numerical integration was carried out by using the package `FIESTA` (see [40]). The checks were done for several values of the variable y.

The explicit expression of the MIs depends on the chosen normalization of the integration measure. The integration on the loop momenta is normalized as follows

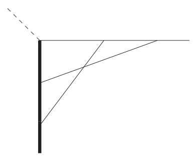
$$\int \mathfrak{D}^d k = \frac{1}{C(d)} \left(\frac{\mu^2}{m^2}\right)^{\frac{(d-4)}{2}} \int \frac{d^d k}{(4\pi^2)^{\frac{(d-2)}{2}}} \,, \tag{A.1}$$

where $C(d)$ is defined in eq. (3.16). In eq. (A.1) μ stands for the 't Hooft mass of dimensional regularization. The integration measure in eq. (A.1) is chosen in such a way that the one-loop massive tadpole becomes

$$\int \mathfrak{D}^d k \, \frac{1}{k^2+m^2} = \frac{m^2}{(d-2)(d-4)} \,. \tag{A.2}$$

In the expressions below, $\mathcal{K}$ is a rational number (its numerical value is $\mathcal{K} = 3.32812 \pm 0.00002 \sim 213/64$), while $a_4 = \mathrm{Li}_4(1/2) = 0.51747906\ldots$. $\zeta(2)$ and $\zeta(3)$ are the Riemann ζ function evaluated in 2 and 3 respectively: $\zeta(2) = 1.6449341\ldots$, $\zeta(3) = 1.2020569\ldots$.

The expressions of the MIs are the following.



$$= \frac{1}{m^4(1+y)^2} \sum_{i=-4}^{0} A_i \, (d-4)^i + \mathcal{O}(d-4) \,, \tag{A.3}$$

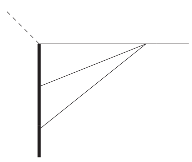
$$A_{-4} = \frac{1}{12} \,, \tag{A.4}$$

$$A_{-3} = \frac{1}{6} H(-1;y) \,, \tag{A.5}$$

$$A_{-2} = -\frac{7}{48}\zeta(2) + \frac{1}{3} H(-1,-1;y) \,, \tag{A.6}$$

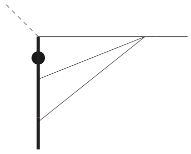
$$A_{-1} = \frac{89}{96}\zeta(3) - \frac{7}{24}\zeta(2)H(-1;y) + \frac{2}{3}H(-1,-1,-1;y) \,, \tag{A.7}$$

$$\begin{aligned} A_0 = & -\frac{2}{5}\zeta^2(2)\mathcal{K} + \frac{65}{48}\zeta(3)H(-1;y) - \frac{7}{12}\zeta(2)H(-1,-1;y) + \frac{4}{3}H(-1,-1,-1,-1;y) \\ & -\frac{1}{2}H(-1,0,0,-1;y) \,. \end{aligned} \tag{A.8}$$



$$= \frac{1}{m^2(1+y)} \, B_0 + \mathcal{O}(d-4) \,, \tag{A.9}$$

$$\begin{aligned} B_0 = & \frac{27}{160}\zeta^2(2) + \frac{3}{16}\zeta(2)H(0,-1;y) - \frac{1}{16}H(0,-1,0,-1;y) + \frac{1}{8}H(0,0,-1,-1;y) \\ & -\frac{1}{16}H(0,0,0,-1;y) \,. \end{aligned} \tag{A.10}$$

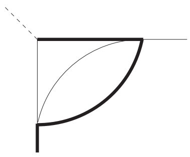


$$= \frac{1}{m^4(1+y)} \sum_{i=-1}^{1} C_i\,(d-4)^i + \mathcal{O}(d-4)^2\,, \tag{A.11}$$

$$C_{-1} = \frac{1}{32}\zeta(2)\,, \tag{A.12}$$

$$C_0 = \frac{1}{64}\zeta(3) + \frac{1}{16}\zeta(2)H(-1;y) + \frac{1}{16}H(0,-1,-1;y) - \frac{1}{16}H(0,0,-1;y)\,, \tag{A.13}$$

$$\begin{aligned} C_1 = & -\frac{9}{80}\zeta^2(2) + \frac{1}{32}\zeta(3)H(-1;y) - \frac{3}{32}\zeta(2)H(0,-1;y) + \frac{1}{8}\zeta(2)H(-1,-1;y) \\ & +\frac{1}{8}H(-1,0,-1,-1;y) - \frac{1}{8}H(-1,0,0,-1;y) + \frac{3}{16}H(0,-1,-1,-1;y) \\ & -\frac{1}{16}H(0,-1,0,-1;y) - \frac{9}{32}H(0,0,-1,-1;y) + \frac{5}{32}H(0,0,0,-1;y)\,. \end{aligned} \tag{A.14}$$



$$= \sum_{i=-2}^{1} D_i\,(d-4)^i + \mathcal{O}(d-4)^2\,, \tag{A.15}$$

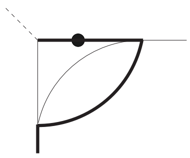
$$D_{-2} = \frac{1}{8}\,, \tag{A.16}$$

$$D_{-1} = -\frac{5}{16} + \frac{(1+y)}{8y}H(-1;y)\,, \tag{A.17}$$

$$\begin{aligned} D_0 = & \frac{19}{32} + \frac{1}{16}\zeta(2) - \frac{5}{16}H(-1;y) + \frac{3}{16}H(-1,-1;y) - \frac{1}{16}H(0,-1;y) \\ & +\frac{1}{y}\left[-\frac{5}{16}H(-1;y) + \frac{3}{16}H(-1,-1;y) - \frac{1}{8}H(0,-1;y)\right] + \frac{1}{(1+y)}\left[\frac{5}{64}\zeta(3) \right. \\ & -\frac{3}{16}\zeta(2)\ln(2) - \frac{3}{16}\zeta(2)H(-2;y) - \frac{1}{16}H(-2,-1,-1;y) - \frac{1}{16}H(0,-1,-1;y) \\ & \left. +\frac{1}{16}H(0,0,-1;y)\right]\,, \end{aligned} \tag{A.18}$$

$$\begin{aligned} D_1 = & -\frac{65}{64} - \frac{7}{128}\zeta(3) - \frac{5}{32}\zeta(2) - \frac{3}{32}\zeta(2)\ln(2) + \frac{19}{32}H(-1;y) + \frac{5}{32}\zeta(2)H(-1;y) \\ & -\frac{3}{32}\zeta(2)H(-2;y) + \frac{5}{32}H(0,-1;y) - \frac{15}{32}H(-1,-1;y) - \frac{1}{32}H(-2,-1,-1;y) \\ & +\frac{5}{16}H(-1,-1,-1;y) - \frac{1}{8}H(-1,0,-1;y) - \frac{3}{32}H(0,-1,-1;y) + \frac{1}{16}H(0,0,-1;y) \\ & +\frac{1}{y}\left[\frac{19}{32}H(-1;y) + \frac{3}{32}\zeta(2)H(-1;y) - \frac{15}{32}H(-1,-1;y) + \frac{5}{16}H(0,-1;y) \right. \\ & \left. +\frac{5}{16}H(-1,-1,-1;y) - \frac{3}{16}H(-1,0,-1;y) - \frac{3}{16}H(0,-1,-1;y) + \frac{1}{8}H(0,0,-1;y)\right] \end{aligned}$$

$$
+\frac{1}{(1+y)}\Big[-\frac{1}{96}\ln^4(2)-\frac{5}{128}\zeta(3)+\frac{3}{32}\zeta(2)\ln(2)+\frac{1}{16}\zeta(2)\ln^2(2)+\frac{33}{640}\zeta^2(2)-\frac{1}{4}a_4
-\Big(\frac{5}{64}\zeta(3)-\frac{3}{16}\zeta(2)\ln(2)\Big)H(-1;y)+\Big(\frac{7}{32}\zeta(3)+\frac{3}{32}\zeta(2)\Big)H(-2;y)
-\frac{3}{32}\zeta(2)H(0,-1;y)+\frac{3}{16}\zeta(2)H(-1,-2;y)-\frac{5}{32}\zeta(2)H(-2,-1;y)
+\frac{1}{32}H(-2,-1,-1;y)+\frac{1}{32}H(0,-1,-1;y)-\frac{1}{32}H(0,0,-1;y)
-\frac{3}{16}H(-2,-1,-1,-1;y)+\frac{1}{16}H(-2,-1,0,-1;y)+\frac{1}{16}H(-1,-2,-1,-1;y)
+\frac{1}{16}H(-1,0,-1,-1;y)-\frac{1}{16}H(-1,0,0,-1;y)-\frac{3}{16}H(0,-1,-1,-1;y)
+\frac{1}{8}H(0,-1,0,-1;y)+\frac{1}{8}H(0,0,-1,-1;y)-\frac{3}{32}H(0,0,0,-1;y)\Big]\,. \qquad \text{(A.19)}
$$



$$
=\frac{1}{m^2}\sum_{i=-1}^{2}E_i\,(d-4)^i+\mathcal{O}(d-4)^3\,, \qquad \text{(A.20)}
$$

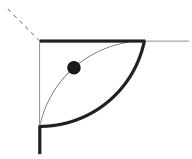
$$
E_{-1}=-\frac{1}{8y}H(-1;y)\,, \qquad \text{(A.21)}
$$

$$
E_0=\frac{1}{8y}\Big[H(-1;y)-\frac{3}{2}H(-1,-1;y)+H(0,-1;y)\Big]
-\frac{1}{16(2+y)}\Big[3\zeta(2)+H(-1,-1;y)\Big]\,, \qquad \text{(A.22)}
$$

$$
E_1=\frac{1}{y}\Big[-\frac{1}{8}H(-1;y)-\frac{3}{32}\zeta(2)H(-1;y)+\frac{3}{16}H(-1,-1;y)
-\frac{5}{16}H(-1,-1,-1;y)+\frac{3}{16}H(-1,0,-1;y)-\frac{1}{8}H(0,-1;y)+\frac{3}{16}H(0,-1,-1;y)
-\frac{1}{8}H(0,0,-1;y)\Big]+\frac{1}{(1+y)}\Big[-\frac{5}{64}\zeta(3)+\frac{3}{16}\zeta(2)\ln(2)+\frac{3}{16}\zeta(2)H(-2;y)
+\frac{1}{16}H(-2,-1,-1;y)+\frac{1}{16}H(0,-1,-1;y)-\frac{1}{16}H(0,0,-1;y)\Big]+\frac{1}{(2+y)}\Big[\frac{7}{32}\zeta(3)
+\frac{3}{16}\zeta(2)-\frac{5}{32}\zeta(2)H(-1;y)+\frac{1}{16}H(-1,-1;y)-\frac{3}{16}H(-1,-1,-1;y)
+\frac{1}{16}H(-1,0,-1;y)\Big]\,, \qquad \text{(A.23)}
$$

$$
E_2=\frac{1}{y}\Big[\Big(\frac{1}{8}+\frac{9}{128}\zeta(3)+\frac{3}{32}\zeta(2)+\frac{3}{32}\zeta(2)\ln(2)\Big)H(-1;y)+\Big(\frac{1}{8}+\frac{3}{32}\zeta(2)\Big)H(0,-1;y)
-\Big(\frac{3}{16}+\frac{7}{32}\zeta(2)\Big)H(-1,-1;y)+\frac{3}{32}\zeta(2)H(-1,-2;y)-\frac{3}{16}H(-1,0,-1;y)
+\frac{1}{8}H(0,0,-1;y)-\frac{3}{16}H(0,-1,-1;y)+\frac{5}{16}H(-1,-1,-1;y)
$$

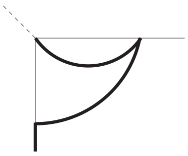
$$
\begin{aligned}
&+\frac{1}{32}H(-1,-2,-1,-1;y)-\frac{9}{16}H(-1,-1,-1,-1;y)+\frac{5}{16}H(-1,-1,0,-1;y)\\
&+\frac{5}{16}H(-1,0,-1,-1;y)-\frac{7}{32}H(-1,0,0,-1;y)+\frac{5}{16}H(0,-1,-1,-1;y)\\
&-\frac{3}{16}H(0,-1,0,-1;y)-\frac{3}{16}H(0,0,-1,-1;y)+\frac{1}{8}H(0,0,0,-1;y)\Big]\\
&+\frac{1}{(1+y)}\Big[\frac{1}{96}\ln^4(2)+\frac{5}{64}\zeta(3)-\frac{3}{16}\zeta(2)\ln(2)-\frac{1}{16}\zeta(2)\ln^2(2)-\frac{33}{640}\zeta^2(2)+\frac{1}{4}a_4\\
&+\Big(\frac{5}{64}\zeta(3)-\frac{3}{16}\zeta(2)\ln(2)\Big)H(-1;y)-\Big(\frac{7}{32}\zeta(3)+\frac{3}{16}\zeta(2)\Big)H(-2;y)\\
&+\frac{3}{32}\zeta(2)H(0,-1;y)+\frac{5}{32}\zeta(2)H(-2,-1;y)-\frac{3}{16}\zeta(2)H(-1,-2;y)\\
&-\frac{1}{16}H(0,-1,-1;y)+\frac{1}{16}H(0,0,-1;y)-\frac{1}{16}H(-2,-1,-1;y)\\
&+\frac{3}{16}H(-2,-1,-1,-1;y)-\frac{1}{16}H(-2,-1,0,-1;y)-\frac{1}{16}H(-1,-2,-1,-1;y)\\
&-\frac{1}{16}H(-1,0,-1,-1;y)+\frac{1}{16}H(-1,0,0,-1;y)+\frac{3}{16}H(0,-1,-1,-1;y)\\
&-\frac{1}{8}H(0,-1,0,-1;y)-\frac{1}{8}H(0,0,-1,-1;y)+\frac{3}{32}H(0,0,0,-1;y)\Big]\\
&+\frac{1}{(2+y)}\Big[-\frac{7}{32}\zeta(3)-\frac{3}{16}\zeta(2)-\frac{45}{128}\zeta^2(2)+\Big(\frac{1}{8}\zeta(3)+\frac{5}{32}\zeta(2)\\
&+\frac{3}{16}\zeta(2)\ln(2)\Big)H(-1;y)-\Big(\frac{1}{16}+\frac{9}{32}\zeta(2)\Big)H(-1,-1;y)+\frac{3}{16}\zeta(2)H(-1,-2;y)\\
&+\frac{3}{16}H(-1,-1,-1;y)-\frac{1}{16}H(-1,0,-1;y)+\frac{1}{16}H(-1,-2,-1,-1;y)\\
&-\frac{7}{16}H(-1,-1,-1,-1;y)+\frac{3}{16}H(-1,-1,0,-1;y)+\frac{5}{32}H(-1,0,-1,-1;y)\\
&-\frac{1}{8}H(-1,0,0,-1;y)\Big]\,. \qquad \text{(A.24)}
\end{aligned}
$$



$$
= \frac{1}{m^2(1+y)}\sum_{i=-1}^{1} F_i\,(d-4)^i + \mathcal{O}(d-4)^2\,, \qquad \text{(A.25)}
$$

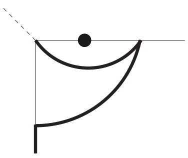
$$
\begin{aligned}
F_{-1} &= \frac{1}{8}\zeta(2)+\frac{1}{8}H(0,-1;y)\,, \qquad \text{(A.26)}\\
F_0 &= -\frac{7}{64}\zeta(3)-\frac{3}{16}\zeta(2)\ln(2)+\frac{1}{8}\zeta(2)H(-1;y)-\frac{3}{16}\zeta(2)H(-2;y)-\frac{1}{16}H(-2,-1,-1;y)\\
&\quad+\frac{1}{8}H(-1,0,-1;y)+\frac{3}{16}H(0,-1,-1;y)-\frac{1}{8}H(0,0,-1;y)\,, \qquad \text{(A.27)}\\
F_1 &= -\frac{1}{96}\ln^4(2)+\frac{1}{16}\zeta(2)\ln^2(2)+\frac{227}{640}\zeta^2(2)-\frac{1}{4}a_4-\frac{3}{16}\zeta(3)H(-1;y)+\frac{7}{32}\zeta(3)H(-2;y)\\
&\quad+\frac{\zeta(2)}{32}[3H(0,-1;y)+4H(-1,-1;y)-5H(-2,-1;y)]-\frac{3}{16}H(-2,-1,-1,-1;y)
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{16}H(-2,-1,0,-1;y)+\frac{1}{8}H(-1,-1,0,-1;y)+\frac{1}{4}H(-1,0,-1,-1;y)\\
&-\frac{3}{16}H(-1,0,0,-1;y)+\frac{5}{16}H(0,-1,-1,-1;y)-\frac{3}{16}H(0,-1,0,-1;y)\\
&-\frac{3}{16}H(0,0,-1,-1;y)+\frac{1}{8}H(0,0,0,-1;y)\,. \qquad \text{(A.28)}
\end{aligned}
$$



$$
= \sum_{i=-2}^{1} G_i\,(d-4)^i + \mathcal{O}(d-4)^2\,, \qquad \text{(A.29)}
$$

$$
\begin{aligned}
G_{-2} &= \frac{1}{8}\,, && \text{(A.30)}\\
G_{-1} &= -\frac{5}{16}\,, && \text{(A.31)}\\
G_0 &= \frac{19}{32}-\frac{1}{16}\zeta(2)-\frac{1}{16}H(0,-1;y)-\frac{1}{8(y+1)}[z3+H(0,0,-1;y)]\,, && \text{(A.32)}\\
G_1 &= -\frac{65}{64}+\frac{3}{32}\zeta(3)+\frac{5}{32}\zeta(2)-\frac{1}{16}\zeta(2)H(-1;y)+\frac{5}{32}H(0,-1;y)-\frac{1}{16}H(-1,0,-1;y)\\
&\quad -\frac{1}{8}H(0,-1,-1;y)+\frac{3}{32}H(0,0,-1;y)+\frac{1}{(1+y)}\Bigg[\frac{1}{16}\zeta(3)+\frac{7}{160}\zeta^2(2)\\
&\quad +\frac{1}{8}\zeta(3)H(-1;y)-\frac{1}{16}\zeta(2)H(0,-1;y)+\frac{1}{16}H(0,0,-1;y)+\frac{1}{8}H(-1,0,0,-1;y)\\
&\quad -\frac{1}{16}H(0,-1,0,-1;y)-\frac{1}{4}H(0,0,-1,-1;y)\Bigg]\,. && \text{(A.33)}
\end{aligned}
$$



$$
= \frac{1}{m^2(1+y)}\sum_{i=-1}^{1} J_i\,(d-4)^i + \mathcal{O}(d-4)^2\,, \qquad \text{(A.34)}
$$

$$
\begin{aligned}
J_{-1} &= \frac{1}{8}\zeta(2)+\frac{1}{8}H(0,-1;y)\,, && \text{(A.35)}\\
J_0 &= -\frac{3}{16}\zeta(3)+\frac{1}{8}\zeta(2)H(-1;y)+\frac{1}{8}H(-1,0,-1;y)+\frac{1}{4}H(0,-1,-1;y)\\
&\quad -\frac{3}{16}H(0,0,-1;y)\,, && \text{(A.36)}\\
J_1 &= \frac{23}{160}\zeta^2(2)-\frac{1}{16}\zeta(3)H(-1;y)+\frac{1}{8}\zeta(2)H(-1,-1;y)+\frac{1}{8}H(-1,-1,0,-1;y)\\
&\quad +\frac{1}{4}H(-1,0,-1,-1;y)-\frac{1}{16}H(-1,0,0,-1;y)+\frac{1}{2}H(0,-1,-1,-1;y)\\
&\quad -\frac{1}{4}H(0,-1,0,-1;y)-\frac{3}{8}H(0,0,-1,-1;y)+\frac{1}{32}H(0,0,0,-1;y)\,. && \text{(A.37)}
\end{aligned}
$$

References

[1] E. Gardi, *On the determination of $|V_{ub}|$ from inclusive semileptonic B decays*, arXiv:0806.4524.

[2] M.A. Shifman and M.B. Voloshin, *Preasymptotic effects in inclusive weak decays of charmed particles*, *Sov. J. Nucl. Phys.* **41** (1985) 120 [*Yad. Fiz.* **41** (1985) 187];
J. Chay, H. Georgi and B. Grinstein, *Lepton energy distributions in heavy meson decays from QCD*, *Phys. Lett.* **B 247** (1990) 399;
I.I.Y. Bigi, N.G. Uraltsev and A.I. Vainshtein, *Nonperturbative corrections to inclusive beauty and charm decays: QCD versus phenomenological models*, *Phys. Lett.* **B 293** (1992) 430 [*Erratum ibid.* **297** (1993) 477] [hep-ph/9207214];
I.I.Y. Bigi, M.A. Shifman, N.G. Uraltsev and A.I. Vainshtein, *QCD predictions for lepton spectra in inclusive heavy flavor decays*, *Phys. Rev. Lett.* **71** (1993) 496 [hep-ph/9304225];
A.F. Falk and M. Neubert, *Second order power corrections in the heavy quark effective theory. 1. Formalism and meson form-factors*, *Phys. Rev.* **D 47** (1993) 2965 [hep-ph/9209268];
A.F. Falk and M. Neubert, *Second order power corrections in the heavy quark effective theory. 2. Baryon form-factors*, *Phys. Rev.* **D 47** (1993) 2982 [hep-ph/9209269];
A.V. Manohar and M.B. Wise, *Inclusive semileptonic B and polarized Λ_b decays from QCD*, *Phys. Rev.* **D 49** (1994) 1310 [hep-ph/9308246];
B. Blok, L. Koyrakh, M.A. Shifman and A.I. Vainshtein, *Differential distributions in semileptonic decays of the heavy flavors in QCD*, *Phys. Rev.* **D 49** (1994) 3356 [*Erratum ibid.* **50** (1994) 3572] [hep-ph/9307247];
T. Mannel, *Operator product expansion for inclusive semileptonic decays in heavy quark effective field theory*, *Nucl. Phys.* **B 413** (1994) 396 [hep-ph/9308262];
M. Neubert, *B physics and CP-violation*, *Int. J. Mod. Phys.* **A 11** (1996) 4173 [hep-ph/9604412].

[3] T. Kinoshita and A. Sirlin, *Radiative corrections to Fermi interactions*, *Phys. Rev.* **113** (1959) 1652;
S.M. Berman, *Radiative corrections to muon and neutron decay*, *Phys. Rev.* **112** (1958) 267.

[4] Y. Nir, *The mass ratio m_c/m_b in semileptonic B decays*, *Phys. Lett.* **B 221** (1989) 184;
M. Jezabek and J.H. Kuhn, *QCD corrections to semileptonic decays of heavy quarks*, *Nucl. Phys.* **B 314** (1989) 1;
M. Trott, *Improving extractions of $|V_{cb}|$ and m_b from the hadronic invariant mass moments of semileptonic inclusive B decay*, *Phys. Rev.* **D 70** (2004) 073003 [hep-ph/0402120];
V. Aquila, P. Gambino, G. Ridolfi and N. Uraltsev, *Perturbative corrections to semileptonic b decay distributions*, *Nucl. Phys.* **B 719** (2005) 77 [hep-ph/0503083].

[5] M.E. Luke, M.J. Savage and M.B. Wise, *Perturbative strong interaction corrections to the heavy quark semileptonic decay rate*, *Phys. Lett.* **B 343** (1995) 329 [hep-ph/9409287];
M.E. Luke, M.J. Savage and M.B. Wise, *Charm mass dependence of the $O(\alpha_s^2 N_f)$ correction to inclusive $B \to X_c e$ anti-electron-neutrino decay*, *Phys. Lett.* **B 345** (1995) 301 [hep-ph/9410387].

[6] P. Ball, M. Beneke and V.M. Braun, *Resummation of running coupling effects in semileptonic B meson decays and extraction of $|V_{cb}|$*, *Phys. Rev.* **D 52** (1995) 3929 [hep-ph/9503492].

[7] A. Czarnecki and K. Melnikov, *Two-loop QCD corrections to $b \to c$ transitions at zero recoil: analytical results*, *Nucl. Phys.* **B 505** (1997) 65 [hep-ph/9703277]; *Two-loop QCD corrections to semileptonic b decays at maximal recoil*, *Phys. Rev. Lett.* **78** (1997) 3630

[hep-ph/9703291]; *Two-loop QCD corrections to semileptonic b decays at an intermediate recoil*, *Phys. Rev.* **D 59** (1999) 014036 [hep-ph/9804215]; *Semileptonic $b \to u$ decays: lepton invariant mass spectrum*, *Phys. Rev. Lett.* **88** (2002) 131801 [hep-ph/0112264].

[8] J. Franzkowski and J.B. Tausk, *$O(\alpha_s^2)$ corrections to $b \to c$ decay at zero recoil*, *Eur. Phys. J.* **C 5** (1998) 517 [hep-ph/9712205].

[9] I.R. Blokland, A. Czarnecki, M. Ślusarczyk and F. Tkachov, *Heavy-to-light decays with a two-loop accuracy*, *Phys. Rev. Lett.* **93** (2004) 062001 [hep-ph/0403221]; *Next-to-next-to-leading order calculations for heavy-to-light decays*, *Phys. Rev.* **D 71** (2005) 054004 [hep-ph/0503039].

[10] A. Pak, I.R. Blokland and A. Czarnecki, *Two-loop QCD corrections to semileptonic b-quark decays near maximum recoil*, *Phys. Rev.* **D 73** (2006) 114009 [hep-ph/0604233].

[11] T. van Ritbergen, *The second order QCD contribution to the semileptonic $b \to u$ decay rate*, *Phys. Lett.* **B 454** (1999) 353 [hep-ph/9903226].

[12] M. Steinhauser and T. Seidensticker, *Second order corrections to the muon lifetime and the semileptonic B decay*, *Phys. Lett.* **B 467** (1999) 271 [hep-ph/9909436].

[13] F. De Fazio and M. Neubert, *$B \to X_u l \bar{\nu}_l$ decay distributions to order α_s*, *JHEP* **06** (1999) 017 [hep-ph/9905351].

[14] U. Aglietti, *Resummed $B \to X_u l\nu$ decay distributions to next-to-leading order*, *Nucl. Phys.* **B 610** (2001) 293 [hep-ph/0104020].

[15] E. Gardi, *On the quark distribution in an on-shell heavy quark and its all-order relations with the perturbative fragmentation function*, *JHEP* **02** (2005) 053 [hep-ph/0501257].

[16] J.R. Andersen and E. Gardi, *Inclusive spectra in charmless semileptonic B decays by dressed gluon exponentiation*, *JHEP* **01** (2006) 097 [hep-ph/0509360].

[17] U. Aglietti, G. Ricciardi and G. Ferrera, *Threshold resummed spectra in $B \to X_u l\nu$ decays in NLO. I*, *Phys. Rev.* **D 74** (2006) 034004 [hep-ph/0507285]; *Threshold resummed spectra in $B \to X_u l\nu$ decays in NLO. II*, *Phys. Rev.* **D 74** (2006) 034005 [hep-ph/0509095]; *Threshold resummed spectra in $B \to X_u l\nu$ decays in NLO. III*, *Phys. Rev.* **D 74** (2006) 034006 [hep-ph/0509271].

[18] P. Gambino, E. Gardi and G. Ridolfi, *Running-coupling effects in the triple-differential charmless semileptonic decay width*, *JHEP* **12** (2006) 036 [hep-ph/0610140].

[19] K. Melnikov, *$O(\alpha_s^2)$ corrections to semileptonic decay $b \to c l \bar{\nu}_l$*, arXiv:0803.0951; A. Pak and A. Czarnecki, *Mass effects in muon and semileptonic $b \to c$ decays*, *Phys. Rev. Lett.* **100** (2008) 241807 [arXiv:0803.0960].

[20] C.W. Bauer, S. Fleming, D. Pirjol and I.W. Stewart, *An effective field theory for collinear and soft gluons: heavy to light decays*, *Phys. Rev.* **D 63** (2001) 114020 [hep-ph/0011336].

[21] C.W. Bauer, D. Pirjol and I.W. Stewart, *Soft-collinear factorization in effective field theory*, *Phys. Rev.* **D 65** (2002) 054022 [hep-ph/0109045].

[22] S.W. Bosch, B.O. Lange, M. Neubert and G. Paz, *Factorization and shape-function effects in inclusive B-meson decays*, *Nucl. Phys.* **B 699** (2004) 335 [hep-ph/0402094].

[23] B.O. Lange, M. Neubert and G. Paz, *Theory of charmless inclusive B decays and the extraction of V_{ub}*, *Phys. Rev.* **D 72** (2005) 073006 [hep-ph/0504071].

JHEP11(2008)065

[24] Z. Ligeti, I.W. Stewart and F.J. Tackmann, *Treating the b quark distribution function with reliable uncertainties*, arXiv:0807.1926.

[25] U. Aglietti et al., *Threshold resummation in $B \to X_c l \nu_l$ decays*, *Phys. Lett.* **B 653** (2007) 38 [arXiv:0707.2010].

[26] P. Gambino, P. Giordano, G. Ossola and N. Uraltsev, *Inclusive semileptonic B decays and the determination of $|V_u b|$*, *JHEP* **10** (2007) 058 [arXiv:0707.2493].

[27] T. Becher and M. Neubert, *Toward a NNLO calculation of the $\bar{B} \to X_s$ gamma decay rate with a cut on photon energy. I: two-loop result for the soft function*, *Phys. Lett.* **B 633** (2006) 739 [hep-ph/0512208].

[28] T. Becher and M. Neubert, *Toward a NNLO calculation of the $\bar{B} \to X_s^+ \gamma$ decay rate with a cut on photon energy. II: two-loop result for the jet function*, *Phys. Lett.* **B 637** (2006) 251 [hep-ph/0603140].

[29] B. Pecjak, *$\bar{B} \to X_u l \bar{\nu} u$ decays at NNLO*, contribution to *ICHEP '08, 34$^{\rm th}$ International Conference on High Energy Physics*, Philadelphia, U.S.A. (2008), arXiv:0810.2921.

[30] R. Bonciani, P. Mastrolia and E. Remiddi, *QED vertex form factors at two loops*, *Nucl. Phys.* **B 676** (2004) 399 [hep-ph/0307295];
U. Aglietti, R. Bonciani, G. Degrassi and A. Vicini, *Two-loop light fermion contribution to Higgs production and decays*, *Phys. Lett.* **B 595** (2004) 432 [hep-ph/0404071];
W. Bernreuther et al., *Two-loop QCD corrections to the heavy quark form factors: the vector contributions*, *Nucl. Phys.* **B 706** (2005) 245 [hep-ph/0406046]; *Two-loop QCD corrections to the heavy quark form factors: axial vector contributions*, *Nucl. Phys.* **B 712** (2005) 229 [hep-ph/0412259]; *Two-loop QCD corrections to the heavy quark form factors: anomaly contributions*, *Nucl. Phys.* **B 723** (2005) 91 [hep-ph/0504190]; *Decays of scalar and pseudoscalar Higgs bosons into fermions: two-loop QCD corrections to the Higgs-quark-antiquark amplitude*, *Phys. Rev.* **D 72** (2005) 096002 [hep-ph/0508254]; *QCD corrections to static heavy quark form factors*, *Phys. Rev. Lett.* **95** (2005) 261802 [hep-ph/0509341];
S. Moch, J.A.M. Vermaseren and A. Vogt, *The quark form factor at higher orders*, *JHEP* **08** (2005) 049 [hep-ph/0507039]; *Three-loop results for quark and gluon form factors*, *Phys. Lett.* **B 625** (2005) 245 [hep-ph/0508055];
T. Gehrmann, T. Huber and D. Maître, *Two-loop quark and gluon form factors in dimensional regularisation*, *Phys. Lett.* **B 622** (2005) 295 [hep-ph/0507061];
R.J. Gonsalves, *Perturbative QCD at high energy colliders*, *Int. J. Mod. Phys.* **E17** (2008) 870 [arXiv:0807.0828].

[31] P. Nogueira, *Automatic Feynman graph generation*, *J. Comput. Phys.* **105** (1993) 279.

[32] S. Laporta and E. Remiddi, *The analytical value of the electron $(g-2)$ at order α^3 in QED*, *Phys. Lett.* **B 379** (1996) 283 [hep-ph/9602417];
S. Laporta, *High-precision calculation of multi-loop Feynman integrals by difference equations*, *Int. J. Mod. Phys.* **A 15** (2000) 5087 [hep-ph/0102033];
F.V. Tkachov, *A theorem on analytical calculability of four loop renormalization group functions*, *Phys. Lett.* **B 100** (1981) 65;
K.G. Chetyrkin and F.V. Tkachov, *Integration by parts: the algorithm to calculate β-functions in 4 loops*, *Nucl. Phys.* **B 192** (1981) 159.

JHEP11(2008)065

[33] A.V. Kotikov, *Differential equations method: new technique for massive Feynman diagrams calculation*, *Phys. Lett.* **B 254** (1991) 158; *Differential equations method: the calculation of vertex type Feynman diagrams*, *Phys. Lett.* **B 259** (1991) 314; *Differential equation method: the calculation of N point Feynman diagrams*, *Phys. Lett.* **B 267** (1991) 123;
E. Remiddi, *Differential equations for Feynman graph amplitudes*, *Nuovo Cim.* **A110** (1997) 1435 [hep-th/9711188];
M. Caffo, H. Czyz, S. Laporta and E. Remiddi, *Master equations for master amplitudes*, *Acta Phys. Polon.* **B29** (1998) 2627 [hep-th/9807119];
M. Caffo, H. Czyz, S. Laporta and E. Remiddi, *The master differential equations for the 2-loop sunrise selfmass amplitudes*, *Nuovo Cim.* **A111** (1998) 365 [hep-th/9805118];
T. Gehrmann and E. Remiddi, *Differential equations for two-loop four-point functions*, *Nucl. Phys.* **B 580** (2000) 485 [hep-ph/9912329];
M. Argeri and P. Mastrolia, *Feynman diagrams and differential equations*, *Int. J. Mod. Phys.* **A 22** (2007) 4375 [arXiv:0707.4037].

[34] A.B. Goncharov, *Multiple polylogarithms, ciclotomy and modular complexes*, *Math. Res. Lett.* **5** (1998) 497;
D.J. Broadhurst, *Massive 3-loop Feynman diagrams reducible to SC* primitives of algebras of the sixth root of unity*, *Eur. Phys. J.* **C 8** (1999) 311 [hep-th/9803091];
E. Remiddi and J.A.M. Vermaseren, *Harmonic polylogarithms*, *Int. J. Mod. Phys.* **A 15** (2000) 725 [hep-ph/9905237];
T. Gehrmann and E. Remiddi, *Numerical evaluation of harmonic polylogarithms*, *Comput. Phys. Commun.* **141** (2001) 296 [hep-ph/0107173];
J. Vollinga and S. Weinzierl, *Numerical evaluation of multiple polylogarithms*, *Comput. Phys. Commun.* **167** (2005) 177 [hep-ph/0410259];
D. Maître, *HPL, a Mathematica implementation of the harmonic polylogarithms*, *Comput. Phys. Commun.* **174** (2006) 222 [hep-ph/0507152]; *Extension of HPL to complex arguments*, hep-ph/0703052.

[35] SemilepFF.txt, file included in the arXiv submission of R. Bonciani and A. Ferroglia, *Two-loop QCD corrections to the heavy-to-light quark decay*, arXiv:0809.4687.

[36] K. Melnikov and T. van Ritbergen, *The three-loop on-shell renormalization of QCD and QED*, *Nucl. Phys.* **B 591** (2000) 515 [hep-ph/0005131].

[37] M. Beneke, T. Huber and X. Li, *Two-loop QCD correction to differential semi-leptonic $b \to u$ decays in the shape-function region*, arXiv:0810.1230.

[38] J.A.M. Vermaseren, *Symbolic manipulation with* FORM, CAN, Amsterdam The Netherlands (1991); *New features of* FORM, math-ph/0010025.

[39] C. Anastasiou and A. Lazopoulos, *Automatic integral reduction for higher order perturbative calculations*, *JHEP* **07** (2004) 046 [hep-ph/0404258].

[40] A.V. Smirnov and M.N. Tentyukov, *Feynman Integral Evaluation by a Sector decomposiTion Approach (FIESTA)*, arXiv:0807.4129.

[41] R. Bonciani, A. Ferroglia, T. Gehrmann, D. Maître and C. Studerus, *Two-loop fermionic corrections to heavy-quark pair production: the quark-antiquark channel*, *JHEP* **07** (2008) 129 [arXiv:0806.2301].

[42] J. Fleischer, M.Y. Kalmykov and A.V. Kotikov, *Two-loop self-energy master integrals on shell*, *Phys. Lett.* **B 462** (1999) 169 [hep-ph/9905249];

JHEP11(2008)065

M. Argeri, P. Mastrolia and E. Remiddi, *The analytic value of the sunrise self-mass with two equal masses and the external invariant equal to the third squared mass*, *Nucl. Phys.* **B 631** (2002) 388 [hep-ph/0202123];
J. Fleischer, A.V. Kotikov and O.L. Veretin, *Analytic two-loop results for selfenergy- and vertex-type diagrams with one non-zero mass*, *Nucl. Phys.* **B 547** (1999) 343 [hep-ph/9808242];
U. Aglietti and R. Bonciani, *Master integrals with one massive propagator for the two-loop electroweak form factor*, *Nucl. Phys.* **B 668** (2003) 3 [hep-ph/0304028].

[43] G. Bell, *Higher order QCD corrections in exclusive charmless B decays*, arXiv:0705.3133; *NNLO vertex corrections in charmless hadronic B decays: imaginary part*, *Nucl. Phys.* **B 795** (2008) 1 [arXiv:0705.3127].

[44] M. Veltman, *Diagrammatica: the path to Feynman rules*, Cambridge Lecture Notes in Physics volume 4, Cambridge University Press, Cambridge U.K. (1994), pg. 284.

[45] H.M. Asatrian, C. Greub and B.D. Pecjak, *NNLO corrections to $B \to X_u l\nu$ in the shape-function region*, arXiv:0810.0987.

[46] G. Bell, *NNLO corrections to inclusive semileptonic B decays in the shape-function region*, arXiv:0810.5695.